Household bargaining, spouses' consumption patterns and the design of commodity taxes

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Abstract

We study the role and structure of commodity taxes in a world where consumption and labor supply decisions are made by couples according to a bargaining procedure between spouses, and where an optimal income tax is also available. When weights (as well as wages) differ across couples, the heterogeneity is multidimensional and the Atkinson and Stiglitz result does not apply. In addition, when the social welfare function is individual based, spouses' social weights may differ from their weights within the couples. This brings about Pigouvian considerations which in themselves may justify commodity taxes. We show that the expressions for the tax rates include Pigouvian and incentive terms. Their roles are most apparent in the case where some goods are consumed exclusively by one of the spouses. Supposing, for instance, that the female spouse has the lower bargaining weight, we find conditions under which the Pigouvian term calls for a subsidization of the "female good", and a taxation of the "male good". The incentive term depends on the distribution of bargaining weights across couples. For instance, for the exclusive consumption case, when the weight of the female spouse increases with wages, the female good tends to be consumed in larger proportion by more productive couples. Consequently, the incentive term makes it a candidate for taxation. In this case the Pigouvian term is mitigated.

Keywords: Couples' taxation, household bargaining, optimal commodity taxation

JEL classification: H21, H31, D10

1 Introduction

This paper brings together two strands of the literature which have hitherto been studied separately. The first one is the role and the design of commodity taxation and the second one is the tax treatment of couples. More precisely, this paper the optimal structure of commodity taxes, in a world where consumption and labor supply decisions are made by couples according to some bargaining procedure between spouses and where an optimal income tax is also available.

The role of commodity taxes is probably one of the most prominent or, at least, one of the oldest issues of taxation policy; see Atkinson (1977). The traditional Ramsey type models which typically advocated nonuniform commodity taxes have received a rather fatal blow by the classic contribution of Atkinson and Stiglitz (1976). In their seminal work, they show that under some conditions—weak separability of preferences in labor supply and goods—an optimal nonlinear income tax is sufficient to implement any incentive compatible Pareto-efficient allocation. In other words, commodity taxes are redundant (or should be uniform). It is by now well understood though that the Atkinson and Stiglitz result has its limitations. In particular, it may not hold under uncertainty (Cremer and Gahvari, 1995) and does not apply under multi-dimensional heterogeneity, for instance, when individuals differ in preferences (Cremer, Gahvari and Ladoux, 1998; and Cremer, Pestieau and Rochet, 2001). When demand behavior is determined by couples according to a bargaining procedure, and weights differ across couples, we are within such a multidimensional setting. In addition, when the social welfare function is individual based, spouses' social weights may differ from their weight within the couple which brings about paternalistic or Pigouvian considerations which in themselves may justify commodity taxes; see Cremer, Gahvari and Ladoux (1998).

The literature on couples' income taxation, though more recent, is also quite substantial. Following the pioneering paper by Boskin and Sheshinski (1987) many authors have studied the taxation of couples both within linear and nonlinear settings.[1] All of these studies concentrate on income taxation and, in particular, the determination of the tax base (with individual or joint taxation as extreme cases). There is typically a single consumption good so that the issue of commodity taxation does not arise. Additionally, most of these papers consider couples as "unitary", and their preferences are represented by what is essentially an individual utility function.[2]

[1] See for instance, Apps and Rees (1988; 1999); Brett (2007); Cremer, Lozachmeur and Pestieau (2012); Kleven, Kreiner and Saez (2009); Schroyen (2003).

[2] An exception is Immervoll, Kleven, Kreiner and Verdelin (2011).

Cremer, Lozachmeur, Maldonado and Roeder (2015), which is the predecessor to the current paper, departs from this unitary couple paradigm and considers bargaining between spouses. They show that this has striking implications for the design of income tax policy. It affects the incentive properties of the nonlinear income tax scheme and introduces Pigouvian considerations into the determination of the spouses' marginal income tax rates. A crucial and rather plausible assumption of their paper is that, while spouses' incomes are publicly observable, the consumption levels of individual spouses are *not* observable. In other words, the allocation of the household's disposable income between spouses is not publicly observable. The Pigouvian elements of the income tax aim at "correcting" the levels of labor supply. This is because from an utilitarian perspective the high-weight spouse tends to work too little. However, while bargaining yields consumption levels for the individual spouses that are also different from the utilitarian ones, the income tax has no leverage on the allocation of the consumption budget within couples. An appropriately designed commodity tax, on the other hand, can affect spouses' budget shares and their (real) consumption budget as long as the male and the female spouse have different tastes. In other words, as long as spending patterns are gender specific, a nonuniform commodity tax provides some partial control of individual consumption levels.

To study this issue, we introduce commodity taxes into a setting which is otherwise similar to Cremer *et al.* (2015). In particular, couples differ in wages and in their bargaining weights. There is an optimal nonlinear income tax scheme based on spouses' incomes which are observable. Individual consumption levels of the different goods are not publicly observable, but anonymous transactions are observable. Consequently, a linear commodity tax is feasible on informational grounds. By now this is the traditional information structure considered in mixed taxation models.

We determine the structure of commodity taxes which maximizes a utilitarian welfare function based on individual utilities. We show that the expressions for the tax rates include Pigouvian and incentive terms. Their respective role is most apparent in the "exclusive" consumption case, where one good is consumed exclusively by the female spouse while another good is exclusively consumed by the male spouse. Assuming that, for instance, the female spouse has the lower bargaining weight, we find conditions under which the Pigouvian term calls for subsidization of the female good and taxation of the male good. The incentive term depends on the distribution of bargaining weights across couples. For the exclusive consumption case, when the weight of the female spouse increases with wages, the female good will tend to be consumed in larger proportion by more productive couples. Consequently, the incentive term makes it a candidate for taxation. Intuitively, under these circumstances a subsidization of the female good would be regressive. The incentive term then mitigates the Pigouvian term and may even reverse it.

The idea that commodity taxes may be used as a device to redistribute *within* households has been explored by Bargain and Donni (2014).[3] However, these authors consider a representative agent (or rather couple) Ramsey setting. Our study differs in two main respects. First, we consider heterogenous couples so that redistribution *between* couples also matters. Second, and most significantly, we depart from the Ramsey setting by considering an optimal income tax. Put differently, we derive the Pareto efficient policy given the information structure. We know from Atkinson and Stiglitz that this changes the nature of the problem in a dramatic way. The role of an extra instrument in this setting is no longer revenue raising, nor redistribution (at least not directly) but to contribute to the screening for the unobservable characteristics. Interestingly, though, some of the results of Bargain and Donni (2014) continue to hold, at least in a qualitative way. Their revenue raising (efficiency) term is no longer present in our expressions. However, the term they refer to as "redistributive" is the counterpart to our Pigouvian terms; both arise because social and private weights differ. The structure of the term is somewhat different, but the main idea that the term calls for a subsidization of the good consumed by the low-weight spouse is already reflected in their expression. However, in our setting intra household redistribution and inter household redistribution may be in conflict. The latter is reflected by the incentive term which has no counterpart in the Ramsey setting.

The remainder of the paper is structured as follows. Section 2 describes the economic framework and analyzes the couple's optimization problem. Section 3 determines the optimal tax policy. An in depth analysis of the optimal tax structure is given by Sections 4 and 5. Specifically, Section 4 analyzes the Pigouvian expressions while Section 5 investigates the incentive term in more detail. Section 6 summarizes and concludes.

2 The couple

Consider a population with $i = 1, ..., n$ couples. The proportion of couple i is π^i. Members of the couple are indexed by the subscript $g = f, m$. Each spouse in couple i supplies ℓ_g^i units of labor earning a wage rate w_g^i. The mating pattern is such that spouses' wages are positively

[3] Backlow and Ray (2003) consider a related problem. They present an empirical analysis which shows how the spouses' respective spending behavior can be used to design a tax reform.

correlated and couples are ordered such that $w_g^i < w_g^{i+1}$. In other words, a higher index refers to a couple in which *both* spouses have a higher wage. Consequently, there is a single level of w_f associated with each level of w_m. The difference in wages between spouses may differ across couples. Gross earnings are given by $y_g^i = w_g^i \ell_g^i$; they are publicly observable for each spouse. With this information, a nonlinear income tax $T(y_f^i, y_m^i)$ is available. The utility of a spouse g in a couple of type i is given by

$$U_g^i = u_g\left(X_g^i\right) - v\left(\ell_g^i\right),$$

where $X_g^i = \left\{x_{g1}^i, ..., x_{gk}^i, ...x_{gK}^i\right\}$ is the K-dimensional consumption vector of spouse $g = f, m$ in couple i. Technologies are linear so that producer prices are given and normalized at one. Individual consumption levels are not observable but anonymous transactions are so that linear (proportional) taxes can be levied on the consumption goods. The consumer prices of goods are given by $p_k = 1 + t_k$ where t_k is the per unit tax levied on good k. Without loss of generality we can fix the tax rate on one of the goods at zero, and we set $t_1 = 0$ so that $p_1 = 1$. Let $p = (p_1, \ldots, p_k, \ldots p_K) = (1, \ldots, p_k, \ldots p_K)$ denote the vector of consumer prices. Labor disutility, v, satisfies $v' > 0$ and $v'' > 0$, while u_g is strictly increasing and concave.

Couples act cooperatively that is, they maximize the weighted sum of spouses' utilities. The weights attached to the female and male spouse in couple i denoted by α_f^i and α_m^i sum up to two, $\alpha_f^i + \alpha_m^i = 2$. We assume that these weights, which reflect the bargaining power of each spouse, are *exogenously* given but may differ between couples.

For our analysis it is convenient to think about the couple as solving a three-stage optimization problem. In a first stage spouses choose their labor supplies and thus their gross income levels, y_f^i and y_m^i, which determine the couple's after tax income I^i:

$$I^i = y_f^i + y_m^i - T(y_f^i, y_m^i).$$

Next, the net income I^i is allocated between spouses so that $I^i = c_f^i + c_m^i$, where c_g^i is the expenditure of spouse g. We assume that the shares of income devoted to the individual spouses are not publicly observable. Finally, each spouse g chooses its consumption bundles given c_g^i.

We solve this three-stage optimization problem by backward induction. Though fairly standard, this exercise is necessary to derive some expressions which will be useful to simplify and interpret the different components for the optimal tax rates studied in Section 3 below.

2.1 Stage 3: consumption vectors

At this stage the y_g^i's and c_g^i's are given. Given the separability of utility, labor supplies are of no direct relevance for the choice of the consumption vector. Spouse g solves

$$\begin{aligned} \max_{X_g^i} \quad & u_g\left(X_g^i\right) \\ \text{s.t.} \quad & \sum_{k=1}^{K} p_k x_{gk}^i \le c_g^i. \end{aligned}$$

Denoting the Lagrange multipliers associated with the budget constraint by δ_g^i, the first order conditions (FOCs) are given by

$$\frac{\partial u_g(X_g^i)}{\partial x_{gk}^i} = \delta_g^i p_k, \quad k = 1, \ldots, K, \quad g = f, m, \quad i = 1, \ldots, n.$$

The resulting demand functions are denoted by $x_{gk}^i(p, c_g^i)\ \forall\ k$. Substituting in the utility function U_g^i yields spouse g's indirect utility function

$$V_g^i(p, c_g^i) = u_g\left(x_{g1}^i\left(p, c_g^i\right), \ldots x_{gK}^i\left(p, c_g^i\right)\right).$$

These are completely standard Marshallian demand and indirect utility functions which satisfy all traditional properties we know from micro theory, including Roy's identity and the Slutsky equation. In particular, note that

$$\delta_g^i = \frac{\partial V_g^i(p, c_g^i)}{\partial c_g^i} = \frac{\partial u_g\left(X_g^i\right)}{\partial x_{g1}^i}. \tag{1}$$

2.2 Stage 2: consumption shares

In stage 2, the couple determines each spouse's consumption share. Recall that I^i denotes the household's disposable (after tax) income. For any bundle $\left(I^i, y_f^i, y_m^i\right)$ couple i solves

$$\max_{c_g^i} \quad W^i = \sum_{g=f,m} \alpha_g^i \left[V_g^i\left(p, c_g^i\right) - v\left(\frac{y_g^i}{w_g^i}\right)\right] \tag{2}$$

$$\text{s.t.} \quad \sum_{g=f,m} c_g^i \le I^i. \tag{3}$$

Substituting the budget constraint into the objective function and differentiating yields the following FOC

$$\frac{\partial W^i}{\partial c_m^i} = \alpha_m^i \delta_m^i - \alpha_f^i \delta_f^i = 0. \tag{4}$$

This equation, along with the budget constraint (3) defines the male's and female's consumption levels as functions of their family income, and the price vector p: $c_g^i\left(p, I^i\right)$. The second order condition (SOC) is negative and given by

$$SOC = \alpha_m^i \frac{\partial^2 V_m^i(p, c_m^i)}{(\partial c_m^i)^2} + \alpha_f^i \frac{\partial^2 V_f^i(p, c_f^i)}{(\partial c_f^i)^2} < 0. \tag{5}$$

Differentiating equation (4) with respect to I^i and p_k yields:

$$\frac{\partial c_m^i(p, I^i)}{\partial I^i} = \frac{\alpha_f^i \frac{\partial^2 V_f^i\left(p, c_f^i\right)}{(\partial c_f^i)^2}}{SOC} > 0, \tag{6}$$

$$\frac{\partial c_m^i(p, I^i)}{\partial p_k} = -\frac{\alpha_m^i \frac{\partial^2 V_m^i\left(p, c_m^i\right)}{\partial c_m^i \partial p_k} - \alpha_f^i \frac{\partial^2 V_f^i\left(p, c_f^i\right)}{\partial c_f^i \partial p_k}}{SOC} \lesseqgtr 0, \tag{7}$$

and

$$\frac{\partial c_f^i(p, I^i)}{\partial I^i} = \frac{\alpha_m^i \frac{\partial^2 V_m^i\left(p, c_m^i\right)}{(\partial c_m^i)^2}}{SOC} > 0, \tag{8}$$

$$\frac{\partial c_f^i(p, I^i)}{\partial p_k} = -\frac{\alpha_f^i \frac{\partial^2 V_f^i\left(p, c_f^i\right)}{\partial c_f^i \partial p_k} - \alpha_m^i \frac{\partial^2 V_m^i\left(p, c_m^i\right)}{\partial c_m^i \partial p_k}}{SOC} = -\frac{\partial c_m^i(p, I^i)}{\partial p_k} \lesseqgtr 0. \tag{9}$$

That is, a spouse's expenditure increases in the couple's disposable income while its reaction to price changes is indeterminate. Obviously we have $\partial c_g^i(p, I^i)/\partial \alpha_g^i > 0$.

To simplify notation let us define

$$\widehat{V}_g^i(p, I^i) \equiv V_g^i(p, c_g^i(p, I^i)), \tag{10}$$

as the indirect sub-utility for spouse g and

$$\chi_{gk}^i\left(p, I^i\right) \equiv x_{gk}^i\left(p, c_g^i\left(p, I^i\right)\right) \tag{11}$$

as the good-k Marshallian demand function of spouse g. Both variables are a function of prices p and disposable household income I^i.

Three properties of the couple's optimal allocation of consumption will be useful for our analysis. First, given $\left(I^i, y_f^i, y_m^i\right)$ the optimal allocation of consumption depends only on overall income I^i and on the weights $\left(\alpha_f^i, \alpha_m^i\right)$ but not on each spouse's labor supply and gross income $\left(y_f^i, y_m^i\right)$. This is due to the separability of utility between consumption and labor. Second, note that

$$\sum_{g=f,m} \frac{\partial c_g^i\left(p, I^i\right)}{\partial I^i} = 1. \tag{12}$$

In words, when a couple's income increases by one dollar so does the sum of their total consumption. Third, the welfare change of an income increase for couple i is given by

$$\frac{\partial W^i}{\partial I^i} = \alpha_f^i \delta_f^i \frac{\partial c_f^i(p, I^i)}{\partial I^i} + \alpha_m^i \delta_m^i \frac{\partial c_m^i(p, I^i)}{\partial I^i},$$

which using equations (4) and (12) yields

$$\frac{\partial W^i}{\partial I^i} = \alpha_f^i \delta_f^i = \alpha_m^i \delta_m^i. \tag{13}$$

2.3 Stage 1: labor supplies

In stage 1, the couple chooses labor supplies. Since we are not aiming at characterizing the optimal income tax scheme, this stage is of no direct relevance to our problem. Consequently, we restrict ourselves to stating the problem which is given by

$$\max_{I^i, y_g^i} \quad W^i = \sum_{g=f,m} \alpha_g^i \left[\widehat{V}_g^i \left(p, I^i\right) - v\left(\frac{y_g^i}{w_g^i}\right)\right] \tag{14}$$

$$\text{s.t.} \quad \sum_{g=f,m} y_g^i - T\left(y_m^i, y_f^i\right) - I^i \geq 0. \tag{15}$$

In words, both spouses choose their labor supplies, taking into account the tax function and the solution of the subsequent stages. The solution is essentially identical (with some change in notation) to that described in Subsection 3.1 of Cremer *et al.* (2015).

3 Optimal tax policy

Throughout the paper we take a paternalistic approach and consider the utilitarian optimum based on equal weights between husband and wife, $\alpha_f^i = \alpha_m^i \; \forall \; i$. The welfare function is thus given by

$$\mathcal{W} = \sum_{i=1}^{n} \pi^i \sum_{g=f,m} \left[\widehat{V}_g^i \left(p, I^i\right) - v\left(\frac{y_g^i}{w_g^i}\right)\right]. \tag{16}$$

Recall that while each spouse's (before tax) income y_g^i is observable, and the distribution of types is common knowledge, productivities, labor supplies and the spouses' individual consumption levels are not publicly observable. To be more precise, neither the spouses' consumption shares c_g^i, nor their respective consumption vectors are observable.

Under the considered information structure the tax instruments include a possibly nonlinear income tax scheme, $T^i \equiv T(y_f^i, y_m^i)$, which can be positive or negative. And since anonymous transactions are observable consumption goods can be taxed in a linear way. This information framework is the one typically considered in mixed taxation models; see, *e.g.*, Christiansen (1984) and Cremer and Gahvari (1997).

With the considered information structure feasible allocations must satisfy the following incentive constraints

$$\sum_{g=f,m} \alpha_g^i \left[\widehat{V}_g^i \left(p, I^i\right) - v\left(\frac{y_g^i}{w_g^i}\right) \right] \geq \sum_{g=f,m} \alpha_g^i \left[\widehat{V}_g^i \left(p, I^b\right) - v\left(\frac{y_g^b}{w_g^i}\right) \right] \quad \forall \;\; i \neq b. \tag{17}$$

That is, any type-i couple must be prevented from mimicking any type-b couple. In addition, the resource constraint

$$\sum_{i=1}^{n} \pi^i \left[\sum_{g=f,m} y_g^i - I^i + \sum_{l=2}^{K} (p_l - 1) \sum_{g=f,m} \chi_{gl}^i \left(p, I^i\right) \right] \geq 0 \tag{18}$$

must hold.[4]

The optimal feasible utilitarian allocation is then obtained by maximizing (16) subject to the constraints (17) and (18). The Lagrangian $\mathcal{L}$ can be written as

$$\begin{aligned} \mathcal{L} = & \sum_{i=1}^{n} \pi^i \sum_{g=f,m} \left[\widehat{V}_g^i \left(p, I^i\right) - v\left(\frac{y_g^i}{w_g^i}\right) \right] \\ & + \sum_{i=1}^{n} \sum_{b=1, b\neq i}^{n} \lambda_{ib} \left\{ \sum_{g=f,m} \alpha_g^i \left[\widehat{V}_g^i \left(p, I^i\right) - v\left(\frac{y_g^i}{w_g^i}\right) \right] - \sum_{g=f,m} \alpha_g^i \left[\widehat{V}_g^i \left(p, I^b\right) - v\left(\frac{y_g^b}{w_g^i}\right) \right] \right\} \\ & + \mu \sum_{i=1}^{n} \pi^i \left[\sum_{g=f,m} y_g^i - I^i + \sum_{l=2}^{K} (p_l - 1) \sum_{g=f,m} \chi_{gl}^i \left(p, I^i\right) \right], \end{aligned} \tag{19}$$

where $\mu > 0$ is the Lagrange multiplier of the resource constraint while $\lambda_{ib} \geq 0$ is the Lagrange multiplier associated with the self-selection constraint from a type-i to a type-b couple. Throughout the paper we assume that only downward incentive constraints are binding. In other words, when $\lambda_{ib} > 0$ we always have $i > b$.[5] The first order conditions with respect to I^i and $p_k \; \forall \; k = 2, ...K$ are stated in the Appendix. We show in Appendix A that optimal commodity

[4] We assume that taxation is purely redistributive; there is no exogenous revenue requirement. This does not have an impact on our results.

[5] This assumption is of no relevance for our formal results. In particular, the expressions for the optimal tax rates are valid whatever the pattern of binding incentive constraints. However, it is convenient for the interpretations. In our setting, where couples can be ranked by increasing wages of both spouses, it is in any event a natural assumption especially with a utilitarian welfare function.

taxes satisfy the following system of equations

$$\begin{pmatrix} t_2 \\ \vdots \\ t_K \end{pmatrix} = -\frac{1}{\mu}\Delta^{-1}\begin{pmatrix} \sum_{i=1}^{n}\pi_i \sum_{g=f,m}(1-\alpha_g^i)\left\{\frac{\partial \widehat{V}_g^i\left(p,I^i\right)}{\partial p_2}+\frac{\partial \widehat{V}_g^i\left(p,I^i\right)}{\partial I^i}\sum_{g=f,m}x_{g2}^i\right\} \\ \vdots \\ \sum_{i=1}^{n}\pi_i \sum_{g=f,m}(1-\alpha_g^i)\left\{\frac{\partial \widehat{V}_g^i\left(p,I^i\right)}{\partial p_K}+\frac{\partial \widehat{V}_g^i\left(p,I^i\right)}{\partial I^i}\sum_{g=f,m}x_{gK}^i\right\} \end{pmatrix}$$

$$+\frac{1}{\mu}\Delta^{-1}\begin{pmatrix} \sum_{i=1}^{n}\sum_{b=1,b\neq i}^{n}\lambda_{bi}\alpha_f^b\delta_f^{bi}\left(\sum_{g=f,m}x_{g2}^i-\sum_{g=f,m}x_{g2}^{bi}\right) \\ \vdots \\ \sum_{i=1}^{n}\sum_{b=1,b\neq i}^{n}\lambda_{bi}\alpha_f^b\delta_f^{bi}\left(\sum_{g=f,m}x_{gK}^i-\sum_{g=f,m}x_{gK}^{bi}\right) \end{pmatrix}, \tag{20}$$

where we define $\delta_g^{bi} \equiv \partial V_g(p, c_g^b\left(p, I^i\right))/\partial c_g^b$ and $x_{gk}^{bi} \equiv x_{gk}(p, c_g^b(p, I^i))$ for $g = f, m$. Δ is the aggregate reduced $(K-1)\times(K-1)$ Slutsky matrix given by

$$\Delta = \begin{pmatrix} \sum_{i=1}^{n}\pi_i\sum_{g=f,m}\frac{\partial\tilde{\chi}_{g2}^i}{\partial p_2} & \dots & \sum_{i=1}^{n}\pi_i\sum_{g=f,m}\frac{\partial\tilde{\chi}_{g2}^i}{\partial p_K} \\ \vdots & \vdots & \vdots \\ \sum_{i=1}^{n}\pi_i\sum_{g=f,m}\frac{\partial\tilde{\chi}_{gK}^i}{\partial p_2} & \dots & \sum_{i=1}^{n}\pi_i\sum_{g=f,m}\frac{\partial\tilde{\chi}_{gK}^i}{\partial p_K} \end{pmatrix}. \tag{21}$$

It is "reduced" in the sense that the line and column pertaining to the untaxed good 1 are removed.[6] The $\tilde{}$ is used to denote the Hicksian demands as a function of prices and household disposible income with[7]

$$\frac{\partial\tilde{\chi}_{gl}^i}{\partial p_k} = \frac{\partial\chi_{gl}^i}{\partial p_k}+\frac{\partial\chi_{gl}^i}{\partial I^i}\sum_{g=f,m}x_{gk}^i \qquad l = 2, ..., K. \tag{22}$$

The Slutsky matrix measures the usual deadweight loss of taxation. The first term on the right hand side is the Pigouvian term. It is zero when $\alpha_f^i = \alpha_m^i = 1\ \forall\ i$, but for the rest its interpretation merits closer investigation. The second term on the right hand side of (20) is the incentive term which depends on relative consumption levels of the mimicking and the mimicked couples.

Before analyzing these terms in greater detail note that to derive the expressions in (20) we combine the FOCs with respect to I^i and p_k of the government's problem to calculate the

[6] This matrix Δ is of full rank so that its inverse exists; see Takayama (1985).

[7] These are standard Hicksian demands but for the couple rather than for the individual. They can be properly defined as solution to the couple's expenditure minimization problem.

compensated derivative of the Lagrangean defined by

$$\frac{\partial \mathcal{L}}{\partial p_k}+\sum_{i=1}^{n}\frac{\partial \mathcal{L}}{\partial I^i}\sum_{g=f,m}x_{gk}^i.$$

This amounts to studying the effect of a variation ($\mathrm{d}p_k$,$\mathrm{d}I^i$) such that

$$\mathrm{d}I^i=\mathrm{d}p_k\sum_{g=f,m}x_{gk}^i.$$

This variation leaves the welfare W^i of couple i unaffected because it does not change $\sum_g \alpha_g^i \widehat{V}_g^i\left(p,I^i\right)$.

We shall now study successively the Pigouvian and the incentive terms in expression (20). In the process it is helpful to decompose the tax rate into the incentive and the Pigouvian part, i.e, $t_k=t_k^P+t_k^{IC}\ \forall\ k=2,...K$. As we will discuss in greater detail in the following, the first term on the right hand side in (20) determines t_k^P while the second one determines t_k^{IC}, that is

$$\begin{pmatrix} t_2^P \\ \vdots \\ t_K^P \end{pmatrix}=-\frac{1}{\mu}\Delta^{-1}\begin{pmatrix} \displaystyle\sum_{i=1}^{n}\pi_i\sum_{g=f,m}\left(1-\alpha_g^i\right)\left\{\frac{\partial \widehat{V}_g^i\left(p,I^i\right)}{\partial p_2}+\frac{\partial \widehat{V}_g^i\left(p,I^i\right)}{\partial I^i}\sum_{g=f,m}x_{g2}^i\right\} \\ \vdots \\ \displaystyle\sum_{i=1}^{n}\pi_i\sum_{g=f,m}\left(1-\alpha_g^i\right)\left\{\frac{\partial \widehat{V}_g^i\left(p,I^i\right)}{\partial p_K}+\frac{\partial \widehat{V}_g^i\left(p,I^i\right)}{\partial I^i}\sum_{g=f,m}x_{gK}^i\right\} \end{pmatrix} \tag{23}$$

and

$$\begin{pmatrix} t_2^{IC} \\ \vdots \\ t_K^{IC} \end{pmatrix}=\frac{1}{\mu}\Delta^{-1}\begin{pmatrix} \displaystyle\sum_{i=1}^{n}\sum_{b=1,b\neq i}^{n}\lambda_{bi}\alpha_f^b\delta_f^{bi}\left(\sum_{g=f,m}x_{g2}^i-\sum_{g=f,m}x_{g2}^{bi}\right) \\ \vdots \\ \displaystyle\sum_{i=1}^{n}\sum_{b=1,b\neq i}^{n}\lambda_{bi}\alpha_f^b\delta_f^{bi}\left(\sum_{g=f,m}x_{gK}^i-\sum_{g=f,m}x_{gK}^{bi}\right) \end{pmatrix}. \tag{24}$$

4 The Pigouvian term

To understand this terminology note that equation (23) gives the optimal tax rates in the benchmark case where wages (couples' types) are observable, while individual consumption levels remain unobservable. In that case the incentive constraints are not relevant; all the λ's are zero and $t_k^{IC}=0\ \forall k$. Further observe that when all spouses in all couples receive identical weights, so that $\alpha_f^i=\alpha_m^i=1\ \forall i$, the Pigouvian tax is zero for all goods, *i.e.*, $t_k^P=0\ \forall k$. Consequently, it appears that when wages are observable the only reason to use commodity taxes is for paternalistic reasons that is, to "correct" the allocation of consumption within couples.

If individual consumption levels were observable they could be perfectly controlled through nonlinear commodity taxes, and the first best utilitarian allocation could be implemented, at least as long as types are observable. The linear commodity taxes considered here only offer an imperfect instrument, but as long as the spouses have different preferences we can expect that they play a role in achieving an intra couple allocation that is closer to the utilitarian optimum.

Intuitively, one would expect that the Pigouvian term calls for a lower tax or even a subsidy on the goods which are consumed in a larger proportion by the low-weight spouse. However, as our analysis will show, this simple conjecture may be misleading and neglects some of the effects that are at work. This is because the taxes affect the spouses' relative consumption shares; see equations (7) and (9). This can be illustrated by a simple example. Assume that one good is mainly consumed by the low-weight female spouse. Then, a subsidy on this good does increase her utility for a given level of c_j^i, but since the consumption shares will be adjusted (in a direction which is not *a priori* obvious) the net impact is not necessarily unambiguous.

Before proceeding, it is also useful to recall some of the results obtained by Cremer *et al.* (2015). In that paper commodity taxes were not available. The optimal income tax also included a Pigouvian term but this one was merely intended to correct spouses' labor supplies. The income tax in itself had no direct effect on spouses' relative consumption shares. And the fact that commodity taxes do have an impact on these consumption shares is precisely the main addition of this paper.

4.1 General expression

Recall that the expressions in (20) measure the effect of a variation ($\mathrm{d}p_k$,$\mathrm{d}I^i$) such that $W^i = \sum_g \alpha_g^i \widehat{V}_g^i \left(p, I^i\right)$ is constant for every i. In words, as consumer prices change, the couple's disposable income is adjusted to keep its utility constant. It is important to stress that while this compensation maintains the couple's utility constant, utilities of individual spouses will, in general, *not* be constant. And it is effectively the impact on the individual spouses' utilities which drives our results. To see this, let's consider the Pigouvian term in (23). The expression

$$\frac{\partial \widetilde{V}_g^i}{\partial p_k} \equiv \frac{\partial \widehat{V}_g^i \left(p, I^i\right)}{\partial p_k} + \frac{\partial \widehat{V}_g^i \left(p, I^i\right)}{\partial I^i} \sum_{g=f,m} x_{gk}^i \tag{25}$$

measures the impact of the considered variation on the utility of spouse $g = f, m$ of a given couple i. Since $\mathrm{d}W^i = 0$, we have

$$\alpha_f^i \frac{\partial \widetilde{V}_f^i}{\partial p_k} + \alpha_m^i \frac{\partial \widetilde{V}_m^i}{\partial p_k} = 0, \tag{26}$$

so that $\partial\widetilde{V}_f^i/\partial p_k$ and $\partial\widetilde{V}_m^i/\partial p_k$ are of opposite sign. Solving for $\partial\widetilde{V}_f^i/\partial p_k$ (or $\partial\widetilde{V}_m^i/\partial p_k$) and substituting, the term pertaining to couple i in line k of the vector in the Pigouvian tax in (23) can then be written as

$$\sum_{g=f,m}\left(1-\alpha_g^i\right)\frac{\partial\widetilde{V}_g^i}{\partial p_k}=\left(1-\frac{\alpha_f^i}{\alpha_m^i}\right)\frac{\partial\widetilde{V}_f^i}{\partial p_k}=\left(1-\frac{\alpha_m^i}{\alpha_f^i}\right)\frac{\partial\widetilde{V}_m^i}{\partial p_k}. \tag{27}$$

The above expression is negative when $\partial\widetilde{V}_f^i/\partial p_k<0$ (so that $\partial\widetilde{V}_m^i/\partial p_k>0$) and $\alpha_f^i<\alpha_m^i$, that is when the low-weight spouses is made worse off by the (couple compensated) tax increase. Note that this is equivalent to saying that the high-weight spouse is made better off.

When Δ is diagonal, implying that the (couple) compensated demand of any good k, $\chi_{gk}\left(p,I^i\right)$ does not depend on the prices of the other goods, the Pigouvian term for good k has the same sign as line k of the vector determined by (27).[8] Consequently, it is negative and thus reduces the tax on good k, or it favors a subsidy if a compensated price *increase* for that good makes the low-weight spouse worse off (so that a compensated price *reduction* makes the low-weight spouse better off). This argument concentrates on a single couple. Substituting from (25) and (27) shows that the Pigouvian tax for good k in (23) is given by

$$\begin{pmatrix} t_2^P \\ \vdots \\ t_K^P \end{pmatrix}=-\frac{1}{\mu}\Delta^{-1}\begin{pmatrix} \sum_{i=1}^n \pi_i\left(1-\frac{\alpha_f^i}{\alpha_m^i}\right)\frac{\partial\widetilde{V}_f^i}{\partial p_2}=\sum_{i=1}^n \pi_i\left(1-\frac{\alpha_m^i}{\alpha_f^i}\right)\frac{\partial\widetilde{V}_m^i}{\partial p_2} \\ \vdots \\ \sum_{i=1}^n \pi_i\left(1-\frac{\alpha_f^i}{\alpha_m^i}\right)\frac{\partial\widetilde{V}_f^i}{\partial p_K}=\sum_{i=1}^n \pi_i\left(1-\frac{\alpha_m^i}{\alpha_f^i}\right)\frac{\partial\widetilde{V}_m^i}{\partial p_k} \end{pmatrix}. \tag{28}$$

In other words, the Pigouvian term pleads for a subsidy on good k if a (couple compensated) price increase for that good makes the low-weight spouse *in all couples* worse off.

When Hicksian demands are not independent (so that Δ is not diagonal), couple compenstad cross price effects come on top of the "direct" effect just described which may then be mitigated or reinforced. The results obtained for the independent case, however, remain valid as long as we assume that the indirect (cross-price) effects are not too significant. We summarize this in the following proposition.

Proposition 1 *Assume that Δ is diagonal, implying that the compensated demand of any good does not depend on the prices of the other goods. Consider an increase in p_k, compensated by an increase in I^i to keep each couples' utility constant. The Pigouvian term pleads for a subsidy on good k if such a compensated increase in its price makes the low-weight spouse in all couples*

[8] The diagonal term of the Slutsky matrix is negative, but the term is premultiplied by a negative sign.

worse off. When Hicksian demands are not independent, cross price effects come on top of the "direct" effect just described which may then be mitigated or reinforced.

4.2 Further results

Returning to a single couple, simplifying equation (27) by making use of (25) and Roy's identity yields

$$\frac{\partial \widetilde{V}_g^i}{\partial p_k} = \delta_g^i \left[\frac{\partial c_g^i}{\partial p_k} + \frac{\partial c_g^i}{\partial I^i}(x_{fk}^i + x_{mk}^i) - x_{gk}^i \right]. \tag{29}$$

This expression shows that $\partial \widetilde{V}_g^i / \partial p_k$ is negative when the variation in spouse g's consumption budget induced by the couple's compensated increase of p_k, i.e., $\partial c_g^i/\partial p_k + (\partial c_g^i/\partial I^i)(x_{fk}^i + x_{mk}^i)$, is smaller than the additional expenditure x_{gk}^i required to keep the couple's consumption of good k constant. We now use this expression to develop the conditions determining the sign of the compensated welfare change of spouse g in terms of demand functions.

4.2.1 The change in spouse g's welfare expressed in terms of couple demand functions

The first approach is to express expression (29) as a function of couple compensated demand functions given by $\chi_{gk}^i \left(p, I^i\right)$. To do so, note that we have by definition, for $g = f, m$

$$c_g^i(p, I^i) = \sum_{k=1}^{K} p_k \chi_{gk}^i(p, I^i) \tag{30}$$

so that

$$\begin{aligned} \frac{\partial c_g^i}{\partial p_k} &= \sum_{l=1}^{K} p_l \frac{\partial \chi_{gl}^i(p, I^i)}{\partial p_k} + x_{gk}^i, \\ \frac{\partial c_g^i}{\partial I^i} &= \sum_{l=1}^{K} p_l \frac{\partial \chi_{gl}^i(p, I^i)}{\partial I^i}. \end{aligned}$$

Substituting these two expressions into (29) yield:

$$\frac{\partial \widetilde{V}_g^i}{\partial p_k} = \delta_g^i \left[\sum_{l=1}^{K} p_l \frac{\partial \widetilde{\chi}_{gl}^i}{\partial p_k} \right]. \tag{31}$$

In other words, a couple compensated price increases spouse g's welfare if and only if the induced compensated change of its total expenditures (included the numeraire good) increases.

4.2.2 The change in spouse g's welfare expressed in terms of individual demand functions

We show in Appendix B that the Pigouvian term in equation (20) for good k and couple i can be rearranged as follows:

$$\frac{\partial \widetilde{V}_f^i}{\partial p_k} = \frac{-\delta_f^i \alpha_f^i}{SOC}\left[\sum_{l=2}^{K} \frac{\partial \tilde{x}_{fl}^i}{\partial p_k}\left(\frac{\partial^2 U_f^i}{\partial x_{f1}^i \partial x_{fl}^i} - p_l \frac{\partial^2 U_f^i}{\left(\partial x_{f1}^i\right)^2}\right) - \frac{\alpha_m^i}{\alpha_f^i}\sum_{l=2}^{K} \frac{\partial \tilde{x}_{ml}^i}{\partial p_k}\left(\frac{\partial^2 U_m^i}{\partial x_{m1}^i \partial x_{ml}^i} - p_l \frac{\partial^2 U_m^i}{\left(\partial x_{m1}^i\right)^2}\right)\right] \tag{32}$$

where $\partial^2 U_g^i / \partial x_{g1}^i \partial x_{gl}^i - p_l \partial^2 U_g^i / \left(\partial x_{g1}^i\right)^2 > 0$ if and only if x_l is a normal normal. Recall that SOC is defined by equation (5). The sign of the term in brackets on the RHS of equation (32) does not appear to be unambiguous. The following examples illustrate the factors by which it is determined.

4.3 Specific cases

We now successively present two examples in order to get more insight on the sign of the Pigouvian tax.

4.3.1 3-goods with exclusive consumption

Let us analyze the special case where one of the goods, say good 2, is exclusively consumed by the female spouse while good 3 is consumed only by the male spouse. Further, we assume that both of these goods are normal (positive income elasticity). Formally, we have $x_{m2}^i = x_{f3}^i = 0 \ \forall \ i$. In other words, x_2 does not enter the male spouse's utility, while x_3 is not an argument of the female spouse's utility function. The numeraire good is consumed by both spouses and, for simplicity, we assume that these are the three only goods, *i.e.*, $K = 3$.

In this example, equation (32) simplifies to:

$$\frac{\partial \widetilde{V}_f^i}{\partial p_2} = \frac{-\delta_f^i \alpha_f^i}{SOC} \frac{\partial \tilde{x}_{f2}^i}{\partial p_2}\left(\frac{\partial^2 U_f^i}{\partial x_{f1}^i \partial x_{f2}^i} - p_2 \frac{\partial^2 U_f^i}{\left(\partial x_{f1}^i\right)^2}\right)$$

and

$$\frac{\partial \widetilde{V}_f^i}{\partial p_3} = \frac{\delta_f^i \alpha_m^i}{SOC} \frac{\partial \tilde{x}_{m3}^i}{\partial p_3}\left(\frac{\partial^2 U_m^i}{\partial x_{m1}^i \partial x_{m3}^i} - p_3 \frac{\partial^2 U_m^i}{\left(\partial x_{m1}^i\right)^2}\right).$$

Using S_{kl} for the terms of the Slutzky matrix Δ defined by (21), the optimal Pigouvian taxes are thus given by:

$$\begin{pmatrix} t_2^P \\ t_3^P \end{pmatrix} = -\frac{1}{\mu}\begin{pmatrix} S_{22} & S_{23} \\ S_{32} & S_{33} \end{pmatrix}^{-1}\begin{pmatrix} K_f \\ K_m \end{pmatrix}, \tag{33}$$

where

$$K_f = \sum_{i=1}^{n} \left(1 - \frac{\alpha_f^i}{\alpha_m^i}\right) \frac{\partial \widetilde{V}_f^i}{\partial p_2},$$

$$K_m = \sum_{i=1}^{n} \left(1 - \frac{\alpha_f^i}{\alpha_m^i}\right) \frac{\partial \widetilde{V}_f^i}{\partial p_3}.$$

Using Cramer's rule to solve (33) yields

$$t_2^P = -\frac{\begin{vmatrix} K_f & S_{23} \\ K_m & S_{33} \end{vmatrix}}{\mu D} = \frac{-K_f S_{33} + K_m S_{23}}{\mu D}, \tag{34}$$

$$t_3^P = -\frac{\begin{vmatrix} S_{22} & K_f \\ S_{32} & K_m \end{vmatrix}}{\mu D} = \frac{-K_m S_{22} + K_f S_{32}}{\mu D}, \tag{35}$$

where D is the determinant of the Slutsky matrix. The concavity of spouses' utilities implies $S_{22}, S_{33} < 0$ and $D > 0$ while the sign of $S_{23} = S_{32}$ is ambiguous.[9]

For the sake of illustration we concentrate on the case where $\alpha_f^i < \alpha_h^i \ \forall\, i$ so that the female spouse has the lower weight in all couples. This implies $K_f < 0$ and $K_m > 0$ when x_{f2}^i and x_{m3}^i are normal goods. Consequently, when Hicksian demands are independent ($S_{23} = S_{32} = 0$), we obtain $t_2^P < 0$ and $t_3^P > 0$. In words, the Pigouvian term calls for a subsidy on the female good and a tax on the male good. The results are exactly reversed if the low-weight spouse is male.

Expressions (34) and (35) show that the results obtained for the diagonal case, namely $t_2^P < 0$ and $t_3^P > 0$ are reinforced when $S_{23} = S_{32} < 0$, that is when goods 2 and 3 are (Hicksian) complements for the couple.[10] They *may* be reversed in the case of Hicksian substitutes ($S_{23} = S_{32} > 0$) but this requires that the cross price (substitution) effects outweigh the own substitution effects.

Intuitively, this can be explained as follows. When the female and male good are complements, the demand for the male good increases when the demand for the female good increases. However, since we want to reduce his consumption level and increase her consumption level, we need an even higher tax on the male good and an even higher subsidy on the female good in case the two are complements.

When the goods are substitutes, the expressions become ambiguous, but their sign remains unchanged as long as the cross price effects are sufficiently small (in absolute value and compared to the direct effects). Now, a decrease in the price of the female good will also decrease

[9] Recall that Δ is the reduced Slutsky matrix. The determinant of the full Slutsky matrix would of course be equal to zero.

[10] To understand the relevance of the Hicksian demand, recall that equation (20) is based on a compensated variation in taxes that leaves the couples' utility unchanged.

the demand for the male good. Consequently, the desired adjustments in female and male consumption can be accomplished with a lower subsidy on the female good and a lower tax on the male good than when demands are independent.

We summarize our results of this section in the following proposition

Proposition 2 *Assume that there are only three goods, one of which is exclusively consumed by the female spouse while the other is exclusively consumed by the male spouse. If the two exclusive goods are normal goods and if they are either (Hicksian) complements or have independent Hicksian demands, the Pigouvian term calls for*

(i) a subsidization of the good exclusively consumed by the low-weight spouse and

(ii) a taxation of the good exclusively consumed by the high-weight spouse.

The results continue to apply for the case of Hicksian substitutes as long as the cross price substitution effects are sufficiently small compared to the own substitution effects.

4.3.2 K-goods with separable utility function

Suppose now that that the utility u_g is given by

$$u_g\left(X_g^i\right) = h\left(x_{g1}^i\right) + v_g\left(x_{g2}^i, ...x_{gK}^i\right), \quad g = f, m. \tag{36}$$

In words, the utility function is separable between the numeraire good 1 and the other goods. Additionally, the sub-utility for good one is the same for both spouses, *i.e.*, $h \equiv h_f = h_m$. Suppose further that $h(x_{g1}^i)$ has constant *absolute* risk aversion (CARA), so that

$$A(x_{g1}^i) \equiv -\frac{\partial^2 h^i/(\partial x_{g1}^i)^2}{\partial h^i/\partial x_{g1}^i} = \bar{A}$$

is constant.[11] We show in appendix C that expression (32) yiels

$$\frac{\partial \widetilde{V}_f^i}{\partial p_k} = \frac{\delta_f^i \alpha_f^i \left[\partial^2 h\left(x_{f1}^i\right)/\left(\partial x_{f1}^i\right)^2\right]}{SOC} \left[\sum_{l=2}^K p_l \left(\frac{\partial \tilde{x}_{fl}^i}{\partial p_k} - \frac{\partial \tilde{x}_{ml}^i}{\partial p_k}\right)\right] \tag{37}$$

so that for $\alpha_f^i < \alpha_m^i$ the Pigouvian term pleads for a subsidy on good k if

$$\sum_{l=2}^K p_l \left|\frac{\partial \tilde{x}_{fl}^i}{\partial p_k}\right| > \sum_{l=2}^K p_l \left|\frac{\partial \tilde{x}_{mk}^i}{\partial p_k}\right| \quad \Leftrightarrow \quad t_k^P < 0. \tag{38}$$

[11] This amounts to assuming that

$$h(x_{g1}^i) = \kappa - e^{\bar{A}x_{g1}^i},$$

where κ is a constant.

In this scenario, we thus obtain that if both spouse's have independent Hicksian demands, the Pigouvian term tends to reduce the tax on good k if the demand of the spouse with the lower weight (assumed to be f for the sake of illustration) is "more responsive" to its price. Responsiveness is defined in terms of the slope the Hicksian demand curve in the CARA case. This generalizes our exclusive consumption result; there the female good was not at all consumed by the male so that its price elasticity was zero. The result suggests that it is not the consumption level *per se* which matters but the sensitivity with respect to the price and thus to the tax or subsidy. This is quite intuitive. When $\alpha_f^i < \alpha_m^i$ female consumption levels will be lower than socially optimal. The Pigouvian element in the tax formula then tends to reduce the difference in consumption levels (or more precisely marginal utilities). This pleads for a subsidy on the goods where female consumption is more price responsive.

We summarize this result in the following proposition.

Proposition 3 *Assuming that Δ is diagonal and preferences are separable in the numeraire good and the other goods, the Pigouvian term pleads for a subsidy on good k if the subutility of the numeraire good exhibits CARA and the slope of the Hicksian demand for good k is larger in absolute value for the low-weight spouse.*

5 The incentive term

5.1 General expression

We now turn to the interpretation of t_k^{IC} given by (24). We concentrate on the case where Δ is diagonal (or near diagonal so that cross price effects are negligible, *i.e.*, $S_{kl} = 0$). These terms have a familiar flavor (see *e.g.*, Cremer and Gahvari, 2014) and their sign is essentially determined by the comparison of the consumption levels of the mimicked and mimicking couples.[12] More precisely, the incentive term is positive and tends to increase the tax if the mimicking couple has a larger total consumption of the considered good than the mimicked couple, i.e., $\sum_g x_{gk}^{ib} > \sum_g x_{gk}^i$. In that case the tax relaxes an otherwise binding incentive constraint because it hurts the mimicking couple more than the couple they mimick. Otherwise, it calls for a subsidy or, at least, a lower tax.[13] The interesting question from our perspective is how these terms are affected by the couple's bargaining and, specifically, by the pattern of spouses' bargaining weights. This is the issue to which we now turn.

[12] Which in turn determines the comparison of the marginal rates of substitution between mimicked and mimicking couples.

[13] Remember that the diagonal terms of the Slutzky matrix are negative.

Observe that since preferences are separable between goods and labor supply, the traditional Corlett and Hague considerations (see, e.g., Christiansen, 1984) do not matter. In other words, issues of complementarity with labor are irrelevant. What matters instead are the spouses' preferences and bargaining weights. Note that if both spouses had the same preferences and weights in all couples, the Atkinson and Stiglitz theorem would apply, and there would be no need for commodity taxes.[14]

Note that x^i_{gk} can effectively also be written as $x^i_{gk} = x^i_{gk}(p, c^i_g(p, I^i, \alpha^i))$, where $\alpha^i \in \left\{\alpha^i_f, \alpha^i_m\right\}$ and $\alpha^i_f + \alpha^i_m = 2$. Consequently, we can write:

$$\sum_{g=f,m} x^i_{gk}(p, c^i_g(p, I^i, \alpha^i)) = x^i_{fk}(p, c^i_f(p, I^i, \alpha^i_f)) + x^i_{mk}(p, I^i - c^i_f(p, I^i, \alpha^i_f))$$

so that

$$\frac{\partial \sum_g x^i_{gk}(p, c^i_g(p, I^i, \alpha^i))}{\partial \alpha^i_f} = \left(\frac{\partial x^i_{fk}}{\partial c^i_f} - \frac{\partial x^i_{mk}}{\partial c^i_m}\right)\frac{\partial c^i_f}{\partial \alpha^i_f}. \tag{39}$$

Equation (39) is positive if

$$\partial x^i_{fk}/\partial c^i_f > \partial x^i_{mk}/\partial c^i_m \tag{40}$$

and negative otherwise. Condition (40) is satisfied if the female spouse's consumption of the considered good is more responsive to income than that of the male spouse. In other words, the Engel curve has a higher slope for the female than for the male spouse.

1. If $\partial x^i_{fk}/\partial c^i_f > \partial x^i_{mk}/\partial c^i_m$ for every i, and $\alpha^i_f < \alpha^b_f$ (plausible case in which couple of type b is richer) then

$$\sum_{g=f,m} x^i_{gk} < \sum_{g=f,m} x^{bi}_{gk} \quad \Leftrightarrow \quad t^{IC}_k > 0$$

so that the incentive term calls for a tax on good k. (this case also applies if $\partial x^i_{fk}/\partial c^i_f < \partial x^i_{mk}/\partial c^i_m$ for every i and $\alpha^i_f > \alpha^b_f$).

2. If $\partial x^i_{fk}/\partial c^i_f < \partial x^i_{mk}/\partial c^i_m$ for every i, and $\alpha^i_f < \alpha^b_f$ then

$$\sum_{g=f,m} x^i_{gk} > \sum_{g,fm} x^{bi}_{gk} \quad \Leftrightarrow \quad t^{IC}_k < 0$$

so that the incentive term calls for a subsidy on good k. (this case also applies if $\partial x^i_{fk}/\partial c^i_f > \partial x^i_{mk}/\partial c^i_m$ for every i and $\alpha^i_f > \alpha^b_f$).

We summarize our results in the following proposition.

[14] With equal weights, the Pigouvian term would also vanish.

Proposition 4 *Assume that Δ is diagonal. The incentive term pleads for a tax (subsidy) on good k if*

(i) the low-weight spouse's consumption of good k is more (less) responsive to income changes and the weight of this spouse is increasing in wages.

(ii) the low-weight spouse's consumption of good k is less (more) responsive to income changes and the weight of this spouse is decreasing in wages.

5.2 3-goods with exclusive consumption

To illustrate the above results it is interesting to return to the exclusive consumption case considered in Subsection 4.1. Recall that good 2 is the good exclusively consumed by f while good 3 is the good exclusively consumed by m. We thus have by definition $\partial x^i_{m2}/\partial c^i_m = \partial x^i_{f3}/\partial c^i_f = 0$ and a couple's total consumption of any of these goods is simply that of one of the spouses. Assume that f is the low-weight spouse in all couples, *i.e.*, $\alpha^i_f < \alpha^i_m$.[15] We then know from Subsection 4.1 that the Pigouvian term calls for a subsidy on good 2 and a tax on good 3. These effects are reinforced by the incentive term if α^i_f decreases with w_i, which automatically implies that α^i_m increases with w_i. In that case the mimicking couple will have a lower consumption of the female good and the incentive term also calls for a subsidy.

However, the case where α^i_f decreases does not appear to be the empirically most compelling; see, for instance, Couprie (2007). And when α^i_f increases with wages we get the opposite result so that the incentive term goes against the Pigouvian term. Intuitively, Pigouvian and redistributive considerations then contradict each other. The female good, which ought to be subsidized on Pigouvian grounds is also consumed in larger proportion by high-wage couples (because f has a higher weight there) and this makes it a candidate for taxation on redistributive grounds.

Proposition 5 *Assume that there are only three goods, one of which is exclusively consumed by the female spouse while the other is exclusively consumed by the male spouse. If the two exclusive goods are normal goods and Hicksian cross price effects are sufficiently small, the Pigouvian term is*

(i) reinforced by the incentive term when the bargaining power of the low-weight spouse decrease with wages and

(ii) is dampened when the bargaining power of the low-weight spouse increases with wages.

[15] The opposite case is exactly symmetric.

6 Summary and conclusion

This paper has studied the design of commodity taxes, in a world where consumption and labor supply decisions are made by couples according to a bargaining procedure between spouses, and where an optimal income tax is also available. We have shown that the expressions for the tax rates include Pigouvian and incentive terms. The Pigouvian term arises when a spouse's social weight differs from her weight within the couple. The incentive terms has a familiar flavor in that it depends on the mimicker and mimicked couples' respective consumption levels. Interestingly, though, these differences in consumption levels depend on the spouses respective bargaining weight. In particular, whether the weight of the low weight spouse increases or decreases with wages has been shown to be of crucial importance. The role of the two terms is most apparent in the case where some goods are consumed exclusively by one of the spouses. Supposing, for instance, that the female spouse has the lower bargaining weight, we have found conditions under which the Pigouvian term calls for a subsidization of the "female good", and a taxation of the "male good". However, when the weight of the female spouse increases with wages, the female good tends to be consumed in larger proportion by more productive couples. Consequently, the incentive term makes it a candidate for taxation. In this case the Pigouvian and incentive terms go in opposite directions.

Appendix

A Derivation of expression (20)

First-order conditions. Differentiating $\mathcal{L}$ with respect to I^i and p_k yields (the arguments of some functions are dropped where no confusion can arise)

$$\frac{\partial \mathcal{L}}{\partial I^i} = \pi^i \sum_{g=f,m} \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial I^i} + \sum_{b=1,b\neq i}^{n} \lambda_{ib} \sum_{g=f,m} \alpha_g^i \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial I^i}$$
$$- \sum_{b=1,b\neq i}^{n} \lambda_{bi} \sum_{g=f,m} \alpha_j^b \frac{\partial \widehat{V}_g^b\left(p, I^i\right)}{\partial I^i} + \mu\pi^i \sum_{l=2}^{K} (p_l - 1) \sum_{g=f,m} \frac{\partial \chi_{gl}^i\left(p, I^i\right)}{\partial I^i} - \mu\pi^i = 0, \tag{A1}$$

$$\frac{\partial \mathcal{L}}{\partial p_k} = \sum_{i=1}^{n} \pi_i \sum_{g=f,m} \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial p_k} + \mu \sum_{i=1}^{n} \pi_i \sum_{g=f,m} \left\{ x_{gk}^i + \sum_{l=2}^{K} (p_l - 1) \frac{\partial \chi_{gl}^i\left(p, I^i\right)}{\partial p_l} \right\}$$
$$+ \sum_{i=1}^{n} \left\{ \sum_{b=1,b\neq i}^{n} \lambda_{ib} \sum_{g=f,m} \alpha_g^i \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial p_k} - \sum_{b=1,b\neq i}^{n} \lambda_{bi} \sum_{g=f,m} \alpha_g^b \frac{\partial \widehat{V}_g^b\left(p, I^i\right)}{\partial p_k} \right\} = 0, \tag{A2}$$

where the tax on good 1 is fixed at zero.

Simplification and rearrangement of (A1). Differentiation of the weighted sum of equation (10) with respect to I^i yields

$$\sum_{g=f,m} \alpha_g^i \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial I^i} = \sum_{g=f,m} \alpha_g^i \frac{\partial V_g^i(p, c_g^i(p, I^i))}{\partial c_g^i} \frac{\partial c_g^i}{\partial I^i} = \sum_{g=f,m} \alpha_g^i \delta_g^i \frac{\partial c_g^i\left(p, I^i\right)}{\partial I^i}.$$

Using equations (12) and (13), this implies that

$$\sum_{g=f,m} \alpha_g^i \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial I^i} = \alpha_g^i \delta_g^i. \tag{A3}$$

Proceeding in the same way and using Roy's identity, we have

$$\sum_{g=f,m} \alpha_g^i \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial p_k} = -\alpha_g^i \delta_g^i \sum_{g=f,m} x_{gk}^i, \tag{A4}$$

$$\sum_{g=f,m} \alpha_g^b \frac{\partial \widehat{V}_g^b\left(p, I^i\right)}{\partial I^i} = \alpha_g^b \delta_g^{bi}, \tag{A5}$$

$$\sum_{g=f,m} \alpha_g^b \frac{\partial \widehat{V}_g^b\left(p, I^i\right)}{\partial p_k} = -\alpha_g^b \delta_g^{bi} \sum_{g=f,m} x_{gk}^{bi}, \tag{A6}$$

where we define

$$\delta_g^{bi} \equiv \partial V_g(p, c_g^b\left(p, I^i\right))/\partial c_g^b \quad \text{and} \quad x_{gk}^{bi} \equiv x_{gk}(p, c_g^b(p, I^i)) \quad \text{for} \quad g = f, m.$$

Rewriting equation (A1) as

$$\pi^i \sum_{g=f,m} (1-\alpha_g^i) \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial I^i} + \left(\pi_i + \sum_{b=1,b\neq i}^{n} \lambda_{ib}\right) \sum_{g=f,m} \alpha_g^i \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial I^i}$$
$$- \sum_{b=1,b\neq i}^{n} \lambda_{bi} \sum_{g=f,m} \alpha_g^b \frac{\partial \widehat{V}_g^b\left(p, I^i\right)}{\partial I^i} + \mu\pi^i \sum_{l=2}^{K} (p_l - 1) \sum_{g=f,m} \frac{\partial \chi_{gl}^i}{\partial I^i} - \mu\pi^i = 0,$$

and making use of equations (A3)-(A6) yields

$$\pi^i \sum_{g=f,m} (1-\alpha_g) \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial I^i} + \left(\pi_i + \sum_{b=1,b\neq i}^{n} \lambda_{ib}\right) \alpha_g^i \delta_g^i$$
$$- \sum_{b=1,b\neq i}^{n} \lambda_{bi} \alpha_g^b \delta_g^{bi} + \mu\pi^i \sum_{l=2}^{K} (p_l - 1) \sum_{g=f,m} \frac{\partial \chi_{gl}^i}{\partial I^i} - \mu\pi^i = 0. \tag{A7}$$

Multiplying (A7) by $\sum_g x_{gk}^i$ and summing over i yields

$$\sum_{i=1}^{n} \frac{\partial \mathcal{L}}{\partial I^i} \sum_{g=f,m} x_{gk}^i = \sum_{i=1}^{n} \pi^i \sum_{g=f,m} (1-\alpha_g) \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial I^i} \sum_{g=f,m} x_{gk}^i$$
$$+ \sum_{i=1}^{n} \left(\pi_i + \sum_{b=1,b\neq i}^{n} \lambda_{ib}\right) \alpha_g^i \delta_g^i \sum_{g=f,m} x_{gk}^i - \sum_{i=1}^{n} \sum_{b=1,b\neq i}^{n} \lambda_{bi} \alpha_g^b \delta_g^{bi} \sum_{g=f,m} x_{gk}^i$$
$$+ \mu \sum_{i=1}^{n} \pi^i \sum_{l} (p_l - 1) \sum_{g=f,m} \frac{\partial \chi_{gl}^i}{\partial I^i} \sum_{g=f,m} x_{gk}^i - \sum_{i=1}^{n} \mu\pi^i \sum_{g=f,m} x_{gk}^i = 0. \tag{A8}$$

Simplification and rearrangement of (A2). We now turn to the FOC with respect to prices. We can rearrange equation (A2) as follows

$$\sum_{i=1}^{n} \pi_i \sum_{g=f,m} (1-\alpha_g^i) \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial p_k} + \sum_{i=1}^{n} \pi_i \sum_{g=f,m} \alpha_g^i \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial p_k}$$
$$+ \mu \sum_{i=1}^{n} \pi_i \sum_{g=f,m} \left\{ x_{gk}^i + \sum_{l} (p_l - 1) \frac{\partial \chi_{gl}^i\left(p, I^i\right)}{\partial p_k} \right\}$$
$$+ \sum_{i=1}^{n} \left\{ \sum_{b=1,b\neq i}^{n} \lambda_{ib} \sum_{g=f,m} \alpha_g^i \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial p_k} - \sum_{b=1,b\neq i}^{n} \lambda_{bi} \sum_{g=f,m} \alpha_g^b \frac{\partial \widehat{V}_g^b\left(p, I^i\right)}{\partial p_k} \right\} = 0$$

which using equations (A3)-(A6) implies

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial p_k} =& \sum_{i=1}^{n} \pi_i \sum_{g=f,m} (1-\alpha_g^i) \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial p_k} - \sum_{i=1}^{n} \pi_i \alpha_g^i \delta_g^i \sum_{g=f,m} x_{gk}^i \\ &+ \mu \sum_{i=1}^{n} \pi_i \sum_{g=f,m} \left\{ x_{gk}^i + \sum_{l} (p_l - 1) \sum_{g=f,m} \frac{\partial \chi_{gl}^i}{\partial p_k} \right\} \\ &+ \sum_{i=1}^{n} \left\{ - \sum_{b=1, b\neq i}^{n} \lambda_{ib} \alpha_g^i \delta_g^i \sum_{g=f,m} x_{gk}^i + \sum_{b=1, b\neq i}^{n} \lambda_{bi} \alpha_g^b \delta_g^{bi} \sum_{g=f,m} x_{gk}^{bi} \right\} = 0. \end{aligned} \tag{A9}$$

Derivation of the compensated derivative of the Lagrangean. Next we calculate the compensated derivative of the Lagrangean defined as

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial p_k} + \sum_{i=1}^{n} \frac{\partial \mathcal{L}}{\partial I^i} \sum_{g=f,m} x_{gk}^i =& \sum_{i=1}^{n} \pi_i \sum_{g=f,m} (1-\alpha_g^i) \left[\frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial p_k} + \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial I^i} \sum_{g=f,m} x_{gk}^i \right] \\ &+ \mu \sum_{i=1}^{n} \pi_i \sum_{l} (p_l - 1) \sum_{g=f,m} \left(\frac{\partial \chi_{gl}^i}{\partial p_k} + \frac{\partial \chi_{gl}^i}{\partial I^i} \sum_{g=f,m} x_{gk}^i \right) \\ &+ \sum_{i=1}^{n} \sum_{b=1, b\neq i}^{n} \lambda_{bi} \alpha_g^b \delta_g^{bi} \left(\sum_{g=f,m} x_{gk}^{bi} - \sum_{g=f,m} x_{gk}^i \right) = 0. \end{aligned} \tag{A10}$$

Rearranging (A10) and noting that $t_l = p_l - 1$, we obtain

$$\begin{aligned}\sum_{l=2}^{K} t_l \sum_{i=1}^{n} \pi_i \sum_{g=f,m} \frac{\partial \tilde{\chi}_{gl}^i}{\partial p_k} =& \frac{1}{\mu} \sum_{i=1}^{n} \sum_{b=1, b\neq i}^{n} \lambda_{bi} \alpha_f^b \delta_f^{bi} \left(\sum_{g=f,m} x_{gk}^i - \sum_{g=f,m} x_{gk}^{bi} \right) \\ &- \frac{1}{\mu} \sum_{i=1}^{n} \pi_i \sum_{g=f,m} (1-\alpha_g^i) \left[\frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial p_k} + \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial I^i} \sum_{g=f,m} x_{gk}^i \right] \end{aligned} \tag{A11}$$

for every $k = 1...K$. Rewriting the system of equations (A11) in matrix notation and premultiplying by Δ^{-1} yields expression (20).

B Derivation of equation (32)

Using (29) equation (27) can be written as

$$\begin{aligned}&\sum_{g=f,m} (1-\alpha_g^i) \left\{ \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial p_k} + \frac{\partial \widehat{V}_g^i\left(p, I^i\right)}{\partial I^i} \sum_{g=f,m} x_{gk}^i \right\} = \sum_{g=f,m} (1-\alpha_g^i) \frac{\partial \widehat{V}_g^i}{\partial p_k} = \left(1 - \frac{\alpha_f^i}{\alpha_m^i} \right) \frac{\partial \widetilde{V}_f^i}{\partial p_k} \\ &= \left(1 - \frac{\alpha_f^i}{\alpha_m^i} \right) \delta_f^i \left[\frac{\partial c_f^i\left(p, I^i\right)}{\partial p_k} + \left(\frac{\partial c_f^i\left(p, I^i\right)}{\partial I^i} - 1 \right) x_{fk}^i + x_{mk}^i \frac{\partial c_f^i\left(p, I^i\right)}{\partial I^i} \right] \end{aligned} \tag{A12}$$

With equations (5), (8), (9) and (12), the term in brakets of equation ($A12$) yields:

$$\frac{\partial c_f^i(p,I^i)}{\partial p_k}+\left(\frac{\partial c_f^i(p,I^i)}{\partial I^i}-1\right)x_{fk}^i+x_{mk}^i\frac{\partial c_f^i(p,I^i)}{\partial I^i}$$
$$=-\frac{\alpha_f^i\frac{\partial^2 V_f^i(p,c_f^i)}{\partial c_f^i\partial p_k}-\alpha_m^i\frac{\partial^2 V_m^i(p,c_m^i)}{\partial c_m^i\partial p_k}}{SOC}-\frac{\alpha_f^i\frac{\partial^2 V_f^i(p,c_f^i)}{(\partial c_f^i)^2}}{SOC}x_{fk}^i+x_{mk}^i\frac{\alpha_m^i\frac{\partial^2 V_m^i(p,c_m^i)}{(\partial c_m^i)^2}}{SOC}$$
$$=\frac{-\alpha_f^i}{SOC}\left[\left(\frac{\partial^2 V_f^i(p,c_f^i)}{\partial c_f^i\partial p_k}+x_{fk}^i\frac{\partial^2 V_f^i(p,c_f^i)}{(\partial c_f^i)^2}\right)-\frac{\alpha_m^i}{\alpha_f^i}\left(\frac{\partial^2 V_m^i(p,c_m^i)}{\partial c_m^i\partial p_k}+x_{mk}^i\frac{\partial^2 V_m^i(p,c_m^i)}{(\partial c_m^i)^2}\right)\right]\quad \text{(A13)}$$

We have

$$\frac{\partial V_f^i(p,c_f^i)}{\partial c_f^i}=\frac{\partial u_f\left(c_f^i-\sum_{l=2}p_l x_{fl}^i(p,c_f^i),x_{f2}^i(p,c_f^i),..,x_{fK}^i(p,c_f^i)\right)}{\partial x_{f1}^i}$$
$$\frac{\partial^2 V_f^i(p,c_f^i)}{\partial c_f^i\partial p_k}=-\frac{\partial^2 u_f^i}{\left(\partial x_{f1}^i\right)^2}\left(x_{fk}^i+\sum_{l=2}p_l\frac{\partial x_{fl}^i}{\partial p_k}\right)+\sum_{l=2}^{K}\frac{\partial x_{fl}^i}{\partial p_k}\frac{\partial^2 u_f^i}{\partial x_{f1}^i\partial x_{fl}^i}$$
$$\frac{\partial^2 V_f^i(p,c_f^i)}{(\partial c_f^i)^2}=\frac{\partial^2 u_f^i}{\left(\partial x_{f1}^i\right)^2}\left(1-\sum_{l=1}p_l\frac{\partial x_{fl}^i}{\partial c_f^i}\right)+\sum_{l=1}\frac{\partial x_{fl}^i}{\partial c_f^i}\frac{\partial^2 u_f^i}{\partial x_{f1}^i\partial x_{fl}^i}$$

so that

$$\frac{\partial^2 V_f^i(p,c_f^i)}{\partial c_f^i\partial p_k}+x_{fk}^i\frac{\partial^2 V_f^i(p,c_f^i)}{(\partial c_f^i)^2}=-\frac{\partial^2 u_f^i}{\left(\partial x_{f1}^i\right)^2}\left(x_{fk}^i+\sum_{l=2}p_l\frac{\partial x_{fl}^i}{\partial p_k}\right)+\sum_{l=1}\frac{\partial x_{fl}^i}{\partial p_k}\frac{\partial^2 u_f^i}{\partial x_{f1}^i\partial x_{fl}^i}$$
$$+\frac{\partial^2 u_f^i}{\left(\partial x_{f1}^i\right)^2}x_{fk}^i\left(1-\sum_{l=2}p_l\frac{\partial x_{fl}^i}{\partial c_f^i}\right)+x_{fk}^i\sum_{l=2}\frac{\partial x_{fl}^i}{\partial c_f^i}\frac{\partial^2 u_f^i}{\partial x_{f1}^i\partial x_{fl}^i}$$
$$=-\frac{\partial^2 u_f^i}{\left(\partial x_{f1}^i\right)^2}\sum_{l=2}p_l\left(\frac{\partial x_{fl}^i}{\partial p_k}+x_{fk}^i\frac{\partial x_{fl}^i}{\partial c_f^i}\right)+\sum_{l=2}\frac{\partial^2 u_f^i}{\partial x_{f1}^i\partial x_{fl}^i}\left(\frac{\partial x_{fl}^i}{\partial p_k}+x_{fk}^i\frac{\partial x_{fl}^i}{\partial c_f^i}\right)$$
$$=\sum_{l=2}^{K}\frac{\partial\tilde{x}_{fl}^i}{\partial p_k}\left(\frac{\partial^2 u_f^i}{\partial x_{f1}^i\partial x_{fl}^i}-p_l\frac{\partial^2 u_f^i}{\left(\partial x_{f1}^i\right)^2}\right),\quad \text{(A14)}$$

where

$$\frac{\partial\tilde{x}_{fl}^i}{\partial p_k}=\frac{\partial x_{fl}^i}{\partial p_k}+x_{fk}^i\frac{\partial x_{fl}^i}{\partial c_f^i}.$$

Similarly, we can show that

$$\frac{\partial^2 V_m^i(p,c_m^i)}{\partial c_m^i\partial p_k}+x_{mk}^i\frac{\partial^2 V_m^i(p,c_m^i)}{(\partial c_m^i)^2}=\sum_{l=2}^{K}\frac{\partial\tilde{x}_{ml}^i}{\partial p_k}\left(\frac{\partial^2 u_m^i}{\partial x_{m1}^i\partial x_{ml}^i}-p_l\frac{\partial^2 u_m^i}{\left(\partial x_{m1}^i\right)^2}\right)\quad \text{(A15)}$$

Substituting equations (A14) and (A15) into (A13) and inserting into (A12) yields equation (32) in the text.

C Proof of expressions (37)

When utility is given by (36), equation (32) can be written as

$$\frac{\partial \widetilde{V}_f^i}{\partial p_k} = \frac{-\delta_f^i \alpha_f^i}{SOC}\left[-\frac{\partial^2 h\left(x_{f1}^i\right)}{\left(\partial x_{f1}^i\right)^2}\sum_{l=2}^{K} p_l \frac{\partial \tilde{x}_{fl}^i}{\partial p_k} + \frac{\alpha_m^i}{\alpha_f^i}\frac{\partial^2 h\left(x_{m1}^i\right)}{\left(\partial x_{m1}^i\right)^2}\sum_{l=2}^{K} p_l \frac{\partial \tilde{x}_{ml}^i}{\partial p_k}\right]$$

Factoring out $-\partial^2 h\left(x_{f1}^i\right) / \left(\partial x_{f1}^i\right)^2$ and using $\alpha_f^i \delta_f^i = \alpha_m^i \delta_m^i$, the above equation can be written as

$$\frac{\partial \widetilde{V}_f^i}{\partial p_k} = \frac{\delta_f^i \alpha_f^i \left[\partial^2 h\left(x_{f1}^i\right) / \left(\partial x_{f1}^i\right)^2\right]}{SOC}\left[\sum_{l=2}^{K} p_l \frac{\partial \tilde{x}_{fl}^i}{\partial p_k} - \frac{\frac{\partial^2 h\left(x_{m1}^i\right)}{\left(\partial x_{m1}^i\right)^2}}{\delta_m^i}\frac{\delta_f^i}{\frac{\partial^2 h\left(x_{f1}^i\right)}{\left(\partial x_{f1}^i\right)^2}}\sum_{l=2}^{K} p_l \frac{\partial \tilde{x}_{ml}^i}{\partial p_k}\right].$$

Noting that $\delta_g^i = \partial h(x_{g1}^i)/\partial x_{g1}^i$ and given that h is CARA, the above equation simplifies to

$$\frac{\partial \widetilde{V}_f^i}{\partial p_k} = \frac{\delta_f^i \alpha_f^i \left[\partial^2 h\left(x_{f1}^i\right) / \left(\partial x_{f1}^i\right)^2\right]}{SOC}\sum_{l=2}^{K} p_l \left(\frac{\partial \tilde{x}_{fl}^i}{\partial p_k} - \frac{\partial \tilde{x}_{ml}^i}{\partial p_k}\right)$$

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