

# Measuring Unfair Inequality: Reconciling Equality of Opportunity and Freedom from Poverty \*

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## Abstract

In this paper we propose a family of measures of unfair inequality by reconciling the measurement of inequality of opportunity with the measurement of poverty. In fact, we provide the first work that combines these two principles in a co-equal fashion without granting dominance of one over the other. By acknowledging that fairness judgments are informed by multiple normative principles we also propose a new route to circumvent the downward biases in equality of opportunity based quantifications of fairness. Empirical application of our measure shows important implications on the debate about inequality. First, average unfair inequality doubles when complementing the ideal of an equal opportunity society with poverty aversion. Second, we show that an exclusive focus on top incomes may miss the most important aspects of the picture when assessing income distributions with respect to their fairness characteristics.

**JEL-Codes:** D31; D63; I32

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# 1 Introduction

Rising inequalities have led to intense debates – both in academia and in the public. But why do we care about inequality? Many theories of distributive justice agree that we should not be concerned by inequalities in outcomes such as income and wealth per se, but that we should rather focus on inequalities that are rooted in factors that could not have been influenced by individual choice.<sup>1</sup> In line with philosophical reasoning, preferences for redistribution elicited in surveys and incentivized experiments show that individuals are more willing to accept income differences which are due to effort (or laziness) rather than exogenous circumstances (Fong, 2001; Cappelen et al., 2007; Alesina and Giuliano, 2011; Alesina et al., 2017). A prominent field of research in economics that has been inspired by the distinction between exogenous circumstances and individual effort is the literature on intergenerational mobility (Black and Devereux, 2011; Chetty et al., 2014a; Chetty et al., 2014b). Also the vast research branches on the gender pay gap (Blau and Kahn, 2017; Kleven et al., 2017) and racial disparities (Lang and Lehmann, 2012; Kreisman and Rangel, 2015) are concerned with inequalities that are rooted in factors beyond individual control.

Yet, in spite of its wide acceptance, invoking the notion of individual responsibility alone is insufficient to define fairness (e.g. Konow, 2003; Konow and Schwettmann, 2016). For example, when an outcome is such that it brings deep deprivation to an individual, questions of how it came about seem secondary to the moral imperative of addressing the extremity of the outcome, be it hunger, homelessness, violence or insecurity (Bourguignon et al., 2006).<sup>2</sup> Hence, while outcome differences based on exogenous circumstances imply violations of fairness, the reverse statement does not hold true. To the contrary, in addition to the responsibility criterion there are many reasons why a given outcome distribution could be considered unfair – one of them being the notion of individual need.

In this paper, we propose the first family of measures for unfair inequality that incorporate the principles of equal opportunities (EOp) and freedom from poverty (FfP) in a co-equal fashion. In line with the previous discussion, we therefore take seriously the idea that life outcomes should be orthogonal to exogenous circumstances (EOp) and everybody should have enough to make ends meet (FfP).

Our paper makes several contributions. First, both EOp and FfP each have a vast theoretical and empirical literature. However, attempts to reconcile the two principles are scant. In existing approaches, priority is given to either principle, while the second serves as a mere weighting factor. To the contrary, in our work we treat EOp and FfP as co-equal principles conveying different grounds for compensation. That is, we develop a measure that detects unfairness emanating from unequal opportunities or poverty even if one of our two guiding principles is satisfied. Following the norm-based approach to inequality measurement (Cowell, 1985; Magdalou and Nock, 2011; Almås et al., 2011), our proposed measure is easily interpretable and carries desirable

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<sup>1</sup>A non-comprehensive list of works emphasizing this distinction include Rawls (1971), Sen (1980), Dworkin (1981a, 1981b), Arneson (1989), Cohen (1989), and Lippert-Rasmussen (2001).

<sup>2</sup>Imagine yourself serving on a soup line. The indigents move forwards towards you and you hand out hot soup. But in one case a new piece of information is given to you. You are told that the outcome of the person in front of you was not the result of exogenous circumstances but a lack of effort. Would you then withdraw your soup holding hand because her outcome is morally justifiable according to the responsibility criterion? If not, clearly some other principle is cutting across the power of the equality of opportunity argument.

properties identified in the respective literature branches on the measurement of EOp and FfP. Standard measures of the two principles are nested in our approach, i.e. excluding either FfP or EOp from the list of normative principles yields a measure with sensible properties from the perspective of the remaining principle.

Second, we effectively propose a new route to address the downward bias inherent in empirical measures of inequality of opportunity. This downward bias is a result of the partial observability of individual information (Ferreira and Gignoux, 2011) and has evoked controversy with respect to the usefulness of such lower bound estimates (Kanbur and Wagstaff, 2016). While previous works have addressed these informational constraints by the use of better datasets and the exploitation of alternative econometric techniques (Niehues and Peichl, 2014; Hufe et al., 2017), we cushion this concern by acknowledging that judgments on the fairness of an outcome distribution are informed by multiple normative principles.

Third, our measure has important implications for the debate about inequalities and the design of appropriate policy responses. In particular, we provide two empirical applications of our measure. First, we calculate unfair inequality in a harmonized dataset of 30 European countries. In our baseline estimations we find that unfair inequality doubles when complementing the concern for unequal opportunities with a concern for freedom from poverty. Second, we measure unfair inequality for the US in the time period from 1969 to 2012. We show that the inequality expansion from the early 1980s up until the beginning 1990s was largely due to the expansion of fair inequality. That is, increases in inequality were only weakly related to factors beyond individual control or based on people falling behind the poverty line. To the contrary, since the beginning of the 1990s the majority share of inequality increases must be considered unfair. This result is equally driven by increasing violations of EOp and FfP. Furthermore, we show that unfairness is nearly unrelated to increases of affluence. Therefore, we argue that the wide-spread focus on top-incomes (Atkinson et al., 2011; Piketty et al., 2018) may miss important aspects of the picture – especially when assessing the development of inequality with respect to aspects of fairness.

This paper is organized as follows. In section 2, we clarify the underlying normative principles of EOp and FfP. We continue by outlining our approach towards reconciling both principles into one measure of unfair inequalities in section 3. Section 4 provides empirical evidence on the extent of unfair inequalities for a set of 30 European countries. Moreover, we analyze the development of unfair inequality in the US for the time period 1969-2012. Lastly, section 5 concludes.

## 2 The Principles

**Equality of Opportunity.** Equality of opportunity is a popular concept of fairness that is used to evaluate distributions of various outcomes, including health status (Rosa Dias, 2009; Trannoy et al., 2010), education (Ferreira and Gignoux, 2014; Oppedisano and Turati, 2015) or income (Aaberge et al., 2011; Ferreira and Gignoux, 2011). Following the seminal contributions by Van de gaer (1993), Fleurbaey (1995), and Roemer (1998), there now is a vivid theoretical

and empirical literature that weaves the idea of personal responsibility into inequality research.<sup>3</sup> For opportunity egalitarians inequalities are ethically acceptable to the extent that they are rooted in factors of individual responsibility. To the contrary, they condemn inequalities that follow from factors beyond individual control. Prominent examples of the latter are, for example, the biological sex, the place of birth, or the socioeconomic outcomes of parents. If individual responsibility factors were the sole determinants of the observed outcome distribution, the ideal of EOp would be realized to its full extent.

To operationalize the idea of EOp, the literature draws on the concepts of *circumstances* and *efforts*, where circumstances are those determinants of individual outcomes for which we do not want to hold individuals responsible. Differences in circumstances are typically taken as an indication that people are of a different *type*. To the contrary, efforts belong to the realm of personal responsibility. To the extent that the former rather than the latter are stronger (weaker) determinants of the empirical outcome distribution, society is considered less (more) fair than otherwise. Note that measures of intergenerational mobility (Black and Devereux, 2011; Chetty et al., 2014a; Chetty et al., 2014b) and the gender pay gap (Blau and Kahn, 2017; Kleven et al., 2017) could be interpreted as measures of inequality of opportunity with one circumstance variable, only. Theoretically EOp is underpinned by two fundamental ideas. First, people should be compensated for unequal circumstances. A prominent formulation of this idea is the principle of *ex-ante compensation* which postulates that the value of opportunity sets ought to be equalized across people with differential circumstances. The principle is *ex-ante* in the sense that opportunity sets are evaluated without regard to the individual level of effort exertion. The second idea inherent to the concept of EOp is that people should be appropriately rewarded for differential efforts. While there are again different formulations of this idea, one prominent version is the principle of *utilitarian reward*. Utilitarian reward states that effort should be rewarded so as to maximize the aggregate outcome of people with the same circumstances. This entails that outcome differences between individuals with the same circumstances are a matter of indifference, i.e. that within-type outcome distributions should only be compared by reference to their aggregate means.<sup>4</sup>

A simple way to formulate the equal-opportunity doctrine by reference to ex-ante compensation and utilitarian reward is the following. Consider a set of individuals  $N = \{1, 2, \dots, n\}$  and an associated vector of non-negative incomes  $Y^e = \{y_1^e, y_2^e, \dots, y_n^e\}$ .<sup>5</sup> Furthermore assume that a set of circumstances  $\Omega = \{C^1, C^2, \dots, C^k\}$  and a scalar of effort,  $\theta$ , jointly determine the outcome of interest  $y$ , i.e.  $g : \theta \times \Omega \mapsto \mathbb{R}^+$ . Based on the expressions  $x_i^k$  of all circumstances  $C^k \in \Omega$ , we can partition  $N$  into a set of mutually exclusive types:  $T = \{t_1, t_2, \dots, t_m\}$ . Individuals  $i$  and  $j$  belong to the same type  $t \in T$  if  $x_i^k = x_j^k \forall C^k \in \Omega$ . They belong to different types  $t \in T$  if  $\exists C^k \in \Omega : x_i^k \neq x_j^k$ . From ex-ante compensation we know that the value of opportunity sets needs to be equalized across these types, while utilitarian reward specifies that opportunity sets need to be compared based on type-specific means. We therefore can construct a counterfac-

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<sup>3</sup>See Roemer and Trannoy (2015), Ferreira and Peragine (2016), and Van de gaer and Ramos (2016) for recent overviews.

<sup>4</sup>For a comprehensive discussion of different compensation and reward principles see the works of Fleurbaey and Peragine (2013) and Van de gaer and Ramos (2016).

<sup>5</sup>Note that superscript  $e$  is used to indicate the empirical income distribution. In section 3 we will also introduce a norm income distribution which is indexed by superscript  $r$ .

tual distribution, where each outcome  $y_i^e$  is replaced by the mean realization of the individual’s type mean,  $\mu_i^e$ , to summarize the opportunity structure of  $N$ . This counterfactual distribution is indicative of unequal opportunities since any inequality can be traced back to differential circumstances and thus a non-compliance with the principle of ex-ante compensation.

Beyond theoretical reasoning, there is compelling empirical evidence that people indeed disapprove of inequalities that are rooted in factors beyond individual control. For instance, in a recent work Alesina et al. (2017) use information treatments to show for a set of industrialized countries that policy preferences with respect to taxation and spending on opportunity-equalizing policies are robustly correlated with perceptions of social mobility. The lower social mobility within a society, the more people are willing to remedy existing inequalities by appropriate policy interventions. Faravelli (2007) demonstrates that perceptions of justice tend to more equal distributions when income differences originate from contextual factors that could not have been influenced by individuals. The works of Cappelen et al. (2007), Cappelen et al. (2010), and Krawczyk (2010) confirm that people uphold the equal-opportunity ideal even if it adversely affects their own material interests.

**Freedom from Poverty.** “Who is poor and who is not?” (Alkire and Foster, 2011) is one of the fundamental questions of poverty research. As in the equal-opportunity literature, the insight that poverty cannot be construed in a uni-dimensional fashion – say, by focusing on some definition of household income only – resonates in many works on the measurement of poverty (Aaberge and Brandolini, 2015). Regardless of the specific outcome dimension of interest, the measurement of poverty is implemented through a poverty line  $y_{\min}$  which provides the threshold of deprivation, and a poverty index which aggregates the deprivations below the threshold. As such the population  $N$  can be partitioned into the groups  $P = \{i : y_i^e \leq y_{\min}\}$  and  $R = \{i : y_i^e > y_{\min}\}$ , where the aggregation index is applied to the incomes of the former group.<sup>6</sup> Akin to the literature on EOp, the normative concern for poverty operates on a principle of compensation: People should be compensated to the extent that they fall short of the poverty line. Naturally, the appropriate setting of the poverty line is a widely debated issue in the literature (among others Foster, 1998; Decerf, 2017). All else equal, the more lenient the definition of  $y_{\min}$ , the larger the group  $P$  to which compensation is owed.

There is ample evidence that people are willing to compensate not just on the base of ex-ante opportunity differences but also based on outcome differences among people with identical circumstances. For example, in Cappelen et al. (2013) both stakeholders and impartial spectators tend to equalize outcomes among participants who engaged in risky choices and the differential outcomes of which are the result of luck. Furthermore, the fairness preferences of people appear to be sensitive to individual needs, i.e. the intuition that everybody should have enough to make ends meet irrespective of how the given outcome came about (Konow, 2001; Gaertner and Schwettmann, 2007).

**Reconciling EOp and FfP.** While the respective literatures on equality of opportunity and poverty are immense individually, attempts to reconcile the two concerns are scant. In the

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<sup>6</sup>Widely used indexes in the literature are the members of the family of measures proposed by Foster et al. (1984), which nests the headcount ratio as well as the poverty gap ratio.

current literature, there seem to be at least three ways to think about EOp and FfP in a joint framework. In spite of their merits, however, all three approaches are characterized by important shortcomings when assessing the fairness of empirical distributions with an eye to both EOp and FfP.

First, in analogy to Lefranc et al. (2008, 2009) one could incorporate a concern for poverty into the evaluation of opportunity sets. In fact, Lefranc et al. (2008, 2009) propose to describe opportunity sets by the area under the type-specific generalized Lorenz curve,  $\mu_t^e(1 - G_t)$ , where  $G_t$  is the type-specific Gini coefficient. As a consequence opportunity sets are evaluated by discounting the type-specific average advantage level  $\mu_t^e$  with the type-specific income risk  $G_t$ . To shift the focus from income risk across the entire income distribution to the risk of deprivation, one could also conceive a different weighting of mean-advantage levels by replacing  $G_t$  with some measure of poverty  $P_t$ .<sup>7</sup> Ceteris paribus, the value of opportunity sets would then decrease with the type-specific prevalence of poverty. Note, however, that such an approach would not treat EOp and FfP on an equal footing. To the contrary, it gives priority to the measurement of EOp, whereas FfP is of secondary importance and serves as a mere weighting factor. Therefore, such an approach would magnify (contract) the violations EOp in case of positive (negative) correlations between type-specific opportunity disadvantages and poverty prevalence. However, FfP carries no weight once the prioritized EOp principle is fulfilled. For illustration purposes, imagine an equal-opportunity society of two equally sized types in each of which 50% of the population fall below the poverty threshold. In this case,  $\mu_1^e(1 - P_1) = \mu_2^e(1 - P_2) = \mu(1 - 0.5)$ , i.e. the value of all opportunity sets would equal the population mean. Hence, such a measure would not raise any moral concern in spite of the fact that 50% of the population fall below the poverty threshold.<sup>8</sup>

Secondly, one could approach this question from the perspective of poverty measurement. This is the strategy pursued by Brunori et al. (2013), who propose an opportunity sensitive poverty measure that effectively weighs incomes below the poverty line by the value of the individual's opportunity set. While their approach is agnostic towards the specific criterion to evaluate opportunity sets, the ex-ante utilitarian evaluation is one of the available options. In this case, identical incomes below the poverty line receive less attention the more advantageous the average outcome of the individual's type. However, just as in the first approach their measures does not treat EOp and FfP on an equal footing. The measures are constructed as poverty indices and therefore they are compliant with the focus axiom (Sen, 1976), i.e. they are invariant to income changes among the non-poor. As a consequence, they do not detect any unfairness in societies that are free from poverty but that are characterized by severe violations of EOp among people earning an income above the poverty line. Again, the secondary principle – in this case EOp – carries no weight once the prioritized principle – in this case FfP – is realized.

A third approach proposed in the literature is the construction of “opportunity-deprivation

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<sup>7</sup>Note that this would require the formulation of some *poverty-averse reward* principle which to the best of our knowledge has not been put forward to this date. For further reference, see also Van de gaer and Ramos (2016) who show how the inequality-averse measure proposed by Lefranc et al. (2008, 2009) can be linked to a combination of ex-ante compensation and inequality-averse reward.

<sup>8</sup>Clearly this example draws on the headcount ratio as the poverty criterion that weighs opportunity sets. Without loss of generality, however, similar examples can be constructed for alternative poverty measures.

profiles” (Ferreira and Peragine, 2016), where members of circumstance types are considered opportunity-deprived if their average outcome falls below a pre-specified deprivation threshold. Effectively, this amounts to applying standard poverty measures to types instead of individuals. Hence, this approach identifies the counterfactual distribution of type-means as its object of analysis. As a consequence it is informative for the identification of particularly opportunity deprived types, however, less so when it comes to evaluating the overall fairness of an empirical income distribution.

In this work we treat EOp and FfP as co-equal principles conveying different grounds for compensation. That is, we develop a measure that is able to detect unfairness emanating from unequal opportunities or poverty even if one of the two guiding principles is satisfied. It’s construction will be outlined in the following section.

### 3 Norm-Based Inequality Measurement

For the purpose of constructing our novel measure of unfair inequality, we rely on the norm-based approach towards inequality measurement (Cowell, 1985; Magdalou and Nock, 2011; Almås et al., 2011). Again consider the society  $N = \{1, 2, \dots, n\}$  and the associated vector of non-negative incomes  $Y^e = \{y_1^e, y_2^e, \dots, y_n^e\}$ . The norm-based approach compares the empirical distribution  $Y^e$  to a reference distribution  $Y^r = \{y_1^r, y_2^r, \dots, y_n^r\}$ .<sup>9</sup> For our purposes  $Y^r$  can be interpreted as the distribution of fair incomes in a society the construction of which is informed by a set of normative principles. Thus, the population is characterized by a vector of income pairs  $Y = \{(y_1^e, y_1^r), (y_2^e, y_2^r), \dots, (y_n^e, y_n^r)\}$ . Unfair inequality is now measured by summarizing the differences between  $Y^e$  and  $Y^r$ . Note that standard measures of inequality, such as the Gini index, can also be understood as norm-based measures, in which the norm vector requires perfect equality among all individuals, i.e.  $y_i^r = \frac{1}{N} \sum_i y_i^e \forall i \in N$ .

**Constructing the Norm Vector.** While the empirical distribution  $Y^e$  is given in the data, the norm vector  $Y^r$  must be constructed by reference to a set of normative requirements. Let’s specify  $D$  as the set of all possible income distributions. Consider the following restrictions on  $D$ , which jointly define the set of eligible norm vectors for our purposes:

$$D^1 = \left\{ D : \sum_i y_i^r = \sum_i y_i^e \right\}. \quad (\text{Constant Resources})$$

Since we are concerned with the fair distribution of available resources in a given society, we want to avoid *creatio ex nihilo*. Thus,  $D^1$  is the subset of norm distributions for which the total amount of resources matches its empirical counterpart.

We evaluate opportunity sets by reference to the principles of ex-ante compensation and utilitarian reward. Hence, we are infinitely inequality averse with respect to inequalities between circumstance types, while we are indifferent with respect to inequality within types (Fleurbaey and Peragine, 2013). As a consequence, opportunity sets can be compared by the respective type means. The ideal of an equal-opportunity society is realized if and only if there is equality

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<sup>9</sup>In the following we will use the terms *norm* and *reference* interchangeably.

across types with respect to their average outcomes.  $D^2$  is the subset of distributions for which this criterion is satisfied:

$$D^2 = \left\{ D : \mu_t^r = \frac{1}{N_t} \sum_{i \in t} y_i^r = \frac{1}{N} \sum_i y_i^e = \mu \quad \forall t \in T \right\}. \quad (\text{EOp})$$

We maintain that people have a claim for a minimal level of subsistence regardless of whether their empirical income is a result of pre-determined circumstance, effort or any other form of luck.<sup>10</sup> Thus,  $D^3$  is the subset of distributions for which this ideal is realized:

$$D^3 = \{ D : y_i^r = y_{\min} \quad \forall i \in P \}. \quad (\text{FFP})$$

Note that we can break down this requirement into two parts:  $y_i^r = \frac{1}{N_P} \sum_{i \in P} y_i^r = \mu_P^r \quad \forall i \in P$  and  $\mu_P^r = y_{\min}$ . The first component states that we are infinitely inequality averse with respect to inequalities among the poor – they all have an *equal* claim to a certain level of resources. The second component states that we are infinitely inequality averse with respect to the average shortfall of the poor population from the poverty line – they all have an equal claim to nothing less (but also nothing more) than exactly the *subsistence level*.

With **Constant Resources** and **FFP** in place, it is evident that  $Y^r$  must be constructed by funneling resources from the individuals of  $R$  to the poor population  $P$ . In order to avoid overcharging the rich and pushing them below the subsistence threshold,  $D^4$  yields the subset of distributions for which each rich individual indeed maintains an income above the poverty line:

$$D^4 = \{ D : y_i^r > y_{\min} \quad \forall i \in R \}. \quad (\text{Financing I})$$

Conditional on the satisfaction of **EOp** and **FFP** we have zero inequality aversion with respect to the share of income that exceeds the poverty line. Hence,  $D^5$  denotes the subset of possible distributions for which within-type inequality of excess income above the poverty line remains unaltered:

$$D^5 = \left\{ D : \forall t \in T, \frac{y_i^r - y_{\min}}{y_j^r - y_{\min}} = \frac{y_i^e - y_{\min}}{y_j^e - y_{\min}} \quad \forall i, j \in t \cap R \right\}. \quad (\text{Financing II})$$

The intersection of these subsets,  $\cap_{s=1}^5 D^s$ , yields a singleton which defines our norm distribution. The construction of  $Y^r$  can be conceived by way of the following two-step procedure in which the funds for transfers are raised through linear tax rates: The realization of **FFP** requires bottom-fill until every  $i \in P$  realizes an income equal to  $y_{\min}$ . In order to avoid pushing the rich below the poverty line (**Financing I**) while maintaining inequality among excess incomes above  $y_{\min}$  within types (**Financing II**), it must be the case that the funds to eradicate poverty are raised by a linear tax on the share of income that exceeds the poverty line. With **FFP** satisfied we can now turn to **EOp** in the second step. Clearly, we must leave incomes of the poor intact in order to maintain **FFP**. As a consequence, transfers across types to realize **EOp** are conducted only among the non-poor individuals. Again, the respective taxes and transfers are calculated in a linear fashion on excess incomes above the poverty line in order to assure compliance with

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<sup>10</sup>See Lefranc et al. (2009) and Lefranc and Trannoy (2017) for the notion of luck within the equality of opportunity framework.

## Financing I and Financing II.

In general, the construction of the reference distribution can be summarized as follows:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^r \leq y_{\min} \\ y_i^e [1 - \tilde{y}_i (\tau^{\text{FFP}} + \tau_t^{\text{EOP}}(1 - \tau^{\text{FFP}}))], & \text{otherwise.} \end{cases} \quad (\text{EOp+FFP})$$

where  $\tilde{y}_i = \left( \frac{y_i^e - y_{\min}}{y_i^e} \right)$ ,  $\tau^{\text{FFP}} = \frac{N_P(y_{\min} - \mu_P^e)}{N_R(\mu_R^e - y_{\min})}$  and  $\tau_t^{\text{EOP}} = \frac{\mu_t^e + \frac{N_{P \cap t}}{N_t}(y_{\min} - \mu_{P \cap t}^e) - \tau^{\text{FFP}}(\frac{N_{R \cap t}}{N_t}(\mu_{R \cap t}^e - y_{\min})) - \mu}{\mu_t^e + \frac{N_{P \cap t}}{N_t}(y_{\min} - \mu_{P \cap t}^e) - \tau^{\text{FFP}}(\frac{N_{R \cap t}}{N_t}(\mu_{R \cap t}^e - y_{\min})) - y_{\min}}$ .

We provide a step by step derivation of the implicit tax rates and the ensuing norm vector in Appendix A.

**Measuring Divergences.** Endowed with both  $Y^e$  and  $Y^r$ , one must decide how to aggregate the discrepancies between both vectors into a scalar measure of unfair inequality. Prominent proposals for appropriate divergence measures include the works by Cowell (1985), Almås et al. (2011), and Magdalou and Nock (2011), each of which can be viewed as a generalization of standard inequality measures. The key difference to standard measures of inequality is that these generalized measures do not decrease (increase) with progressive (regressive) transfers from rich (poor) to poor (rich) but rather with transfers that reduce (increase) the aggregate distance between the empirical and the reference distribution. Hence, transfers from poor to rich can be desirable if the empirical income of the poor exceeds its reference value, while the income of the rich falls short of it. Note that the latter requirement is equivalent to the standard Pigou-Dalton principle of transfers if the reference distribution is equivalent to the sample mean  $\mu$ .

Our baseline estimates are presented in terms of the measures proposed by Magdalou and Nock (2011). This choice is guided by two intuitions. First, as opposed to the unfairness Gini (Almås et al., 2011), the measures proposed by Cowell (1985) and Magdalou and Nock (2011) are subgroup-decomposable. Second, the latter classes nest the mean log deviation, i.e. the generalized entropy measure with  $r = 0$ , if we set  $y_i^r = \mu \forall i \in N$ . As such we ensure close proximity to the literature on the measurement of unequal opportunities, in which the use of the mean log deviation is prevalent (among others Ferreira and Gignoux, 2011; Hufe et al., 2017). Robustness checks with respect to the specific choice of the divergence measure, however, including the measures proposed by Cowell (1985) and Almås et al. (2011) are provided in section 4.3.

Magdalou and Nock (2011) propose the following generalization of the general entropy class of inequality measures:<sup>11</sup>

$$D(Y^e || Y^r) = \sum_i [\phi(y_i^e) - \phi(y_i^r) - (y_i^e - y_i^r)\phi'(y_i^r)],$$

where

$$\phi(y) = \begin{cases} -\ln y, & \text{if } r = 0 \\ y \ln y, & \text{if } r = 1 \\ \frac{1}{r(r-1)}y, & \text{otherwise.} \end{cases}$$

<sup>11</sup>We abbreviate this class with  $MN$  in the following.

As in the family of generalized entropy measures,  $r$  is indicative of different value judgments. The higher  $r$ , the higher the valuation of positive divergences of empirical incomes  $y_i^e$  from their respective norm incomes  $y_i^r$ . To the contrary, the lower  $r$  the more weight is attached to shortfalls from  $y_i^r$ . In the baseline we choose  $r = 0$ . This parameterization yields the mean log deviation if the norm vector was equal to the sample mean.<sup>12</sup> Thus, for our baseline estimates of unfair inequality, we aggregate divergences between the reference and empirical distribution as follows:

$$D(Y^e||Y^r) = \frac{1}{N} \sum_i \left[ \ln \frac{y_i^r}{y_i^e} + \frac{y_i^e}{y_i^r} - 1 \right].^{13} \quad (1)$$

Replacing  $y_i^r$  with norm distribution **EOP+FfP** we obtain the following baseline measure of unfair inequality:

$$\begin{aligned} D(Y^e||Y_{\text{EOP+FfP}}^r) &= \frac{1}{N} \sum_{i \in P} \left\{ \ln \frac{y_{\min}}{y_i^e} - \left( \frac{y_{\min} - y_i^e}{y_{\min}} \right) \right\} \\ &+ \frac{1}{N} \sum_{i \in R} \left\{ \ln (1 - \tilde{y}_i (\tau^{\text{FfP}} + \tau_t^{\text{EOP}} (1 - \tau^{\text{FfP}}))) + \frac{\tilde{y}_i (\tau^{\text{FfP}} + \tau_t^{\text{EOP}} (1 - \tau^{\text{FfP}}))}{1 - \tilde{y}_i (\tau^{\text{FfP}} + \tau_t^{\text{EOP}} (1 - \tau^{\text{FfP}}))} \right\}. \quad (\text{Baseline}) \end{aligned}$$

**Properties.** To explore the properties of our measure, we first ask the question of how it behaves if we focus on one of our normative principles only. Ideally excluding either FfP or EOp from our list of normative concerns yields a measure with sensible properties and interpretability from the perspective of opportunity measurement or poverty measurement, respectively.

First, assume our normative evaluation of a given income distribution was blind to FfP. Then,  $y_{\min} = 0$ ,  $N_R = N$  and  $N_{R \cap t} = N_t$ . This leads to  $\tau^{\text{FfP}} = 0$  and  $\tau_t^{\text{EOP}} = \frac{\mu_i^e - \mu}{\mu_t^e}$ . Hence, our norm vector would look as follows:

$$y_i^r = y_i^e \left[ 1 - \left( \frac{\mu_i^e - \mu}{\mu_t^e} \right) \right] = y_i^e \left[ \frac{\mu}{\mu_t^e} \right]. \quad (\text{EOp Only})$$

Using **EOp Only** in equation 1, we see that our measure of unfair inequality collapses to measuring inequality in the distribution of average outcomes by circumstance type. Hence, we obtain the standard ex-ante utilitarian measure of equality of opportunity in which the mean log deviation is applied to a smoothed distribution of type-specific mean incomes:

$$D(Y^e||Y_{\text{EOp}}^r) = \frac{1}{N} \sum_i \ln \frac{\mu}{\mu_t^e}. \quad (2)$$

Reversely, one could ask whether we obtain a poverty measure with sensible properties when focusing on FfP only. Therefore, second, let us assume that our normative evaluation of  $Y^e$  was indifferent to EOp. In this case  $\mu_t^e = \mu$ ,  $N_{R \cap t} = N_R$ ,  $\mu_{R \cap t}^e = \mu_R$  and hence  $\tau_t^{\text{EOP}} = 0$ . As a consequence the norm-vector would be constructed as follows:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e \leq y_{\min} \\ y_i^e [1 - \tilde{y}_i \tau^{\text{FfP}}], & \text{otherwise.} \end{cases} \quad (\text{FfP Only})$$

<sup>12</sup>Sensitivity checks with alternating specifications of  $r$  are provided in section 4.3.

<sup>13</sup>Note that we can scale the measure by  $\frac{1}{N}$  to satisfy the *principle of populations* without further adjustment since we have specified the reference distribution to satisfy **Constant Resources** (Magdalou and Nock, 2011).

Using **FfP Only** in equation 1, we obtain the following measure of unfair inequality:<sup>14</sup>

$$D(Y^e || Y_{\text{FFP}}^r) = \underbrace{\frac{1}{N} \sum_{i \in P} \ln \frac{y_{\min}}{y_i^e}}_{\text{Watts Index}} - \underbrace{\frac{1}{N} \sum_{i \in P} \left( \frac{y_{\min} - y_i^e}{y_{\min}} \right)}_{\text{Poverty Gap}} \quad (3)$$

$$+ \frac{1}{N} \sum_{i \in R} \left\{ \ln (1 - \tilde{y}_i \tau^{\text{FFP}}) + \left( \frac{\tilde{y}_i \tau^{\text{FFP}}}{1 - \tilde{y}_i \tau^{\text{FFP}}} \right) \right\}. \quad (4)$$

As the MN-measures are additively decomposable, we can evaluate unfairness in the truncated distribution of  $Y_P^e = \{y_1^e, y_2^e, \dots, y_{\min}\}$ . Focusing on the contributions of  $i \in P$ , we see that unfairness in this part of the distribution is characterized by the difference between the Watts index (Zheng, 1993) and the poverty gap ratio. Individually, these are well-known measures of poverty, however, also their combination bears a number of desirable properties that have been identified in the literature on poverty measurement (e.g. Ravallion and Chen, 2003). These include *monotonicity* (as opposed for example to the headcount ratio), the *principle of transfers* (as opposed for example to the poverty gap taken as a stand-alone measure) and *additive decomposability*.<sup>15</sup>

We conclude that our measure nests sensible measures of both EOp and FfP. Therefore, it is easily interpretable and carries desirable properties identified in the respective literature on the measurement of either EOp or FfP.

**Comparison to Other Approaches.** Our critique of previous approaches to reconcile EOp and FfP included the fact that there are cases when unfair inequality remains undetected once the prioritized principle is satisfied. So how does our composite measure behave once we live in a society free from deprivation or where individual outcomes cannot be traced to differential circumstances?

First, assume that all individuals earn above the minimum threshold of income, i.e.  $y_i^e > y_{\min}$ . Then,  $N_R = N$  and  $N_{R \cap t} = N_t$ . This leads to  $\tau^{\text{FFP}} = 0$  and  $\tau_t^{\text{EOp}} = \frac{\mu_i^e - \mu}{\mu_i^e - y_{\min}}$ . We would obtain the following norm-vector:

$$y_i^r = y_i^e [1 - \tilde{y}_i \tau_t^{\text{EOp}}]. \quad (\text{Poverty Free})$$

Note the difference to our previous thought experiment, in which we abstracted from FfP altogether. Here our concern for poverty remains intact, which by **Financing I** leads us to calculate opportunity-equalizing transfers across types by reference to the income share that exceeds  $y_{\min}$ . After all we want to avoid pushing non-deprived households into poverty for the sake of realizing EOp. As a consequence, within a given type, the contribution to opportunity equalization across types raises with the proportion of individual income that exceeds  $y_{\min}$ . In contrast to standard ex-ante utilitarian measures of EOp, we are not indifferent to any inequality within types and hence obtain a measure that detects more unfairness in a given outcome distribution.

Second, assume that we live in an equal opportunity society without disparities across different circumstance types. In such a society assuring FfP remains the only normative ground

<sup>14</sup>For the sake of visual clarity we do not spell out the norm income of individuals above the poverty line.

<sup>15</sup>Note that it is unsurprising that we do not obtain a measure of poverty that satisfies the *focus axiom*. Our approach frames poverty as a question of inequality and thus imposes requirements on how the funds to eradicate poverty should be raised (**Financing I** and **Financing II**). Therefore, it is not indifferent to transfers between incomes above the poverty line and thus violates the *focus axiom*.

for policy intervention. In this case,  $\mu_t^e = \mu$ ,  $N_{R\cap t} = N_R$ ,  $\mu_{R\cap t}^e = \mu_R$  and hence  $\tau_t^{\text{EOP}} = 0$ . As a consequence, the norm vector would be constructed exactly as in **FfP Only**. Thus, our measure coincides with the measure constructed in our previous thought experiment, in which we abstracted from the goal of EOp altogether:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e \leq y_{\min} \\ y_i^e [1 - \tilde{y}_i \tau^{\text{FfP}}], & \text{otherwise.} \end{cases} \quad (\text{Equal Opportunity})$$

To conclude: In contrast to previous attempts to reconcile EOp and FfP, our measure detects unfairness in a given outcome distribution even if either of our normative principles is perfectly satisfied.

Table 1 summarizes the measures that have been discussed in the previous paragraphs.

Table 1: Measures

Scenario	Description	Measure
<b>EOp+FfP</b> (Baseline)	$D(Y^e    Y_{\text{EOp+FfP}}^r)$	$\frac{1}{N} \sum_{i \in P} \left\{ \ln \frac{y_{\min}}{y_i^e} - \left( \frac{y_{\min} - y_i^e}{y_{\min}} \right) \right\}$ $+ \frac{1}{N} \sum_{i \in R} \left\{ \ln (1 - \tilde{y}_i (\tau^{\text{FfP}} + \tau_t^{\text{EOP}} (1 - \tau^{\text{FfP}}))) \right\}$ $+ \frac{1}{N} \sum_{i \in R} \left\{ \frac{\tilde{y}_i (\tau^{\text{FfP}} + \tau_t^{\text{EOP}} (1 - \tau^{\text{FfP}}))}{1 - \tilde{y}_i (\tau^{\text{FfP}} + \tau_t^{\text{EOP}} (1 - \tau^{\text{FfP}}))} \right\}$
<b>EOp Only</b>	$D(Y^e    Y_{\text{EOp}}^r)$	$\frac{1}{N} \sum_i \ln \frac{\mu}{\mu_t^e}$
<b>FfP Only</b>	$D(Y^e    Y_{\text{FfP}}^r)$	$\frac{1}{N} \sum_{i \in P} \left\{ \ln \frac{y_{\min}}{y_i^e} - \left( \frac{y_{\min} - y_i^e}{y_{\min}} \right) \right\}$ $+ \frac{1}{N} \sum_{i \in R} \left\{ \ln (1 - \tilde{y}_i \tau^{\text{FfP}}) + \left( \frac{\tilde{y}_i \tau^{\text{FfP}}}{1 - \tilde{y}_i \tau^{\text{FfP}}} \right) \right\}$
<b>Poverty Free</b>	$D(Y^e    Y_{\text{EOp+FfP}}^r)$ if $y_i^e > y_{\min} \forall i \in N$	$\frac{1}{N} \sum_i \left\{ \ln (1 - \tilde{y}_i \tau_t^{\text{EOP}}) + \left( \frac{\tilde{y}_i \tau_t^{\text{EOP}}}{1 - \tilde{y}_i \tau_t^{\text{EOP}}} \right) \right\}$
<b>Equal Opportunity</b>	$D(Y^e    Y_{\text{EOp+FfP}}^r)$ if $\mu_t^e = \mu \forall t \in T$	$\frac{1}{N} \sum_{i \in P} \left\{ \ln \frac{y_{\min}}{y_i^e} - \left( \frac{y_{\min} - y_i^e}{y_{\min}} \right) \right\}$ $+ \frac{1}{N} \sum_{i \in R} \left\{ \ln (1 - \tilde{y}_i \tau^{\text{FfP}}) + \left( \frac{\tilde{y}_i \tau^{\text{FfP}}}{1 - \tilde{y}_i \tau^{\text{FfP}}} \right) \right\}$

**Note:** The displayed measures are constructed based on MN with  $r = 0$ .  $\tilde{y}_i = \left( \frac{y_i^e - y_{\min}}{y_{\min}} \right)$ ,  $\tau^{\text{FfP}} = \frac{NP(y_{\min} - \mu_P^e)}{NR(\mu_R^e - y_{\min})}$  and  $\tau_t^{\text{EOP}} =$

$$\frac{\mu_t^e + \frac{NP\cap t}{N_t}(y_{\min} - \mu_{P\cap t}^e) - \tau^{\text{FfP}} \left( \frac{NR\cap t}{N_t}(\mu_{R\cap t}^e - y_{\min}) \right) - \mu}{\mu_t^e + \frac{NP\cap t}{N_t}(y_{\min} - \mu_{P\cap t}^e) - \tau^{\text{FfP}} \left( \frac{NR\cap t}{N_t}(\mu_{R\cap t}^e - y_{\min}) \right) - y_{\min}}$$

## 4 Empirical Application

### 4.1 Data

To illustrate the suggested measure of unfair inequalities, we draw on two data sources. First, we provide cross-country evidence for 30 European countries in 2010 using the EU Statistics on Income and Living Conditions (EU-SILC). Second, we use the the Panel Study of Income

Dynamics (PSID) to illustrate the development of unfair inequality in the US over the time period 1969-2012.

**EU-SILC.** The 2011 wave of EU-SILC covers 31 European countries.<sup>16</sup> It is a well-researched database for monitoring inequality, poverty and social exclusion in Europe (see for example Atkinson et al. (2017) and the references cited therein), which lends our results to easy comparison with previous works. In particular, we use the 2011 wave as it provides a module on the intergenerational transmission of advantages, which allows us to construct types from circumstance variables.<sup>17</sup> As is common in survey data, incomes are reported for the year preceding the survey, i.e. 2010 in our case.

We follow standard practices from the literature branches on inequality and poverty measurement in setting up the data. We focus on disposable household income adjusted by the OECD-equivalence scale as the outcome of interest.<sup>18</sup> The few observations with  $y_i^e < 0$  are excluded from the analysis. We replace zero incomes by one to avoid sample reductions through logarithmic transformations. To curb the influence of outliers in the lower and the upper part of the income distribution, we winsorize at the 1st and the 99.95th-percentile of the country-specific income distribution. We hold the poverty rate  $y_{\min}$  fixed at the so called European *At-Risk-Of-Poverty Rate* which is drawn at 60% of the country-specific median equivalized disposable household income.<sup>19</sup> Furthermore, we restrict the sample to working age individuals of 25-59 years. To assure the representativeness of the sample all calculations are performed considering personal cross-sectional sample weights. Lastly, note that we exclude Sweden from further analysis due to a sample size of  $N < 1,000$ . As a consequence we obtain a cross-section of 30 countries.

**PSID.** PSID is a panel study that tracks more than 18,000 individuals from approximately 5,000 families in the United States over time. From its inception in 1968 up until today it provides nationally representative information on the US based non-immigrant population including their incomes, labor market histories, background characteristics and living practices. Starting with annual surveys, it has switched to a biennial rhythm in 1997. Analogously to EU-SILC, it is well researched with respect to aspects of inequality and poverty (Niehues and Peichl, 2014; Kimberlin et al., 2016) which allows us to benchmark our results against existing studies. In terms of data preparation we exactly follow the procedure outlined for EU-SILC.

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<sup>16</sup>The sample consists of Austria (AT), Belgium (BE), Bulgaria (BG), Switzerland (CH), Cyprus (CY), Czech Republic (CZ), Germany (DE), Denmark (DK), Estonia (EE), Greece (EL), Spain (ES), Finland (FI), France (FR), Croatia (HR), Hungary (HU), Ireland (IE), Iceland (IS), Italy (IT), Malta (MT), Lithuania (LT), Luxembourg (LU), Latvia (LV), Netherlands (NL), Norway (NO), Poland (PL), Portugal (PT), Romania (RO), Sweden (SE), Slovenia (SI), Slovakia (SK), and Great Britain (UK).

<sup>17</sup>The 2005 wave also comprises a module on the intergenerational transmission of advantages for a sample of 26 European countries. Results for the 2005 wave are available on request.

<sup>18</sup>This is standard in poverty measurement since the notion of deprivation typically refers to individual well-being as approximated by consumption possibilities. To the contrary, individual gross incomes are often used in the measurement of unequal opportunities. However, usually the differences between inequality of opportunity in gross and disposable incomes are small since existing tax-benefit system typically ignore circumstances used to estimate inequality of opportunity. For example, tax rates usually do not include tags (Immonen et al., 1998; Gelber and Weinzierl, 2016) for gender or the education of parents.

<sup>19</sup>The specification of  $y_{\min}$  is a normative assumption in itself and a strong driver of our empirical results. Results for alternative poverty lines are provided in section 4.3.

Equivalized disposable household income is calculated based on the cross-national equivalent file (CNEF) of the PSID. We exclude negative incomes, replace zero incomes by one, and winsorize at the 1st and the 99.95th-percentile of the year-specific income distribution. The poverty line is fixed at 60% of the year-specific median equivalized disposable household income and the sample is restricted to the age corridor 25-59. To assure representativeness, all analyses are conducted using the cross-sectional weights provided by the PSID. All incomes refer to the year prior to the survey. Since CNEF is only available until 2013 and the PSID does not contain important circumstance information in waves prior to 1970, we present results for the time period 1969-2012.

**Circumstances and Types.** For the estimation of inequality of opportunity it is indispensable to partition the population into types. In this work we use four circumstance variables, that are frequently utilized in the empirical literature on equality of opportunity. The first circumstance is the biological sex of the respondent. Second, in EU-SILC we proxy the respondent’s migration background by a binary indicator for whether the respondent lived in her country of birth at time of survey completion. In the US, we replace this circumstance with a racial indicator differentiating among whites and non-whites. Third, we use information on the educational status of the parents. Particularly, we construct types based on whether the highest educated parent of a respondent dropped out of secondary education, attained a secondary school degree or whether the highest educated parent of a respondent attended at least some tertiary education. Lastly, we proxy the occupational status of both parents by grouping them in either elementary occupations, semi-skilled occupations, or top-rank positions.<sup>20</sup> We only retain information on the parent of highest occupational status. As such each of the considered populations is partitioned into a maximum of  $2 * 2 * 3 * 3 = 36$  non-overlapping circumstance types. As illustrated in Tables B.1 and B.2 some country (year) observations fall short of 36 types. This is due to the fact that some combinations of circumstances are appear rather seldom in the data. To give an intuitive example, the combination of the highest educated parent having less than a secondary school degree but occupying a top-rank position in her profession is extremely rare. In order to curb the influence of very small types we only retain those types for which we have a minimum of 20 observations in the respective country-year cell.

**Descriptive Statistics.** Table B.1 shows descriptive statistics for the income distributions in the European sample. In 2010 mean disposable household income was lowest in Romania, Bulgaria and Lithuania ( $\mu < 5,000\text{€}$ ). At the top of the intra-European country distribution we find Luxembourg, Norway and Switzerland the average disposable household incomes of which hover around the mark of 40,000€.<sup>21</sup> To characterize the lower end of the income distributions we draw on three measures of poverty: The headcount ratio ( $P_0$ ), the poverty gap ratio ( $P_1$ ) and the Watts index. The headcount ratio is only sensitive to the number of people falling short of  $y_{\min}$ , while the poverty gap also measures their average distance to the poverty line. Beyond these two properties, the Watts index additionally varies with inequality among the poor. Indeed

<sup>20</sup>Details on the harmonization of classifications within and across datasets are provided in Table C.1 in the Appendix.

<sup>21</sup>Note that the data is not adjusted for purchasing power parity. This has no bearing on our central results since the statistics of interest are both calculated on the country level and scale invariant.

country rankings based on these three measures show some variation. For example, Hungary (Rank 16), Luxembourg (Rank 17) rank fairly bad in terms of the headcount ratio, but perform considerably better in terms of the poverty gap (Ranks 11 and 10). To the contrary, Denmark and Iceland rank 7th and 8th in terms of  $P_0$  but only 18th and 15th in terms of  $P_1$ . This indicates that in the first group of countries there is a relatively high mass of people falling short of  $y_{\min}$ , yet on average this shortfall is relatively small in comparison to other countries. In the two Nordic countries, however, poverty is not very pervasive, but relatively severe on average for those who actually fall behind the deprivation threshold. Rankings according to the Watts index are largely comparable to the poverty gap ratio.

Going beyond the subset  $P$ , we characterize the entire income distribution by means of the Gini index and the mean log deviation. While the Gini index is particularly sensitive to transfers in the middle, the mean log deviation has a normative focus on the lower end of the income distribution. In spite of their different foci, both inequality indices yield rather stable country rankings. According to both measures Norway is the most equal society within our sample. At the upper end of the inequality spectrum we find the Baltic countries of Lithuania and Latvia. The most severe re-ranking occurs for Denmark, that falls back from rank 8 to rank 15 when considering the mean log deviation instead of the Gini index. This suggests that inequality in Denmark is particularly driven by the poor falling short of the population average.

Descriptive statistics for the US are shown in Table B.2. Consistent with other works we observe pronounced increases of both poverty and inequality beginning with the 1980s. These observations are independent of the particular measure we employ. For example, the poverty headcount increased from 11.6% in the first period of observation in 1969 to 12.9% in 1980. This moderate increase is followed by a stronger upwards movement to a level of 16.8% in 1990, 17.3% in 2000 and 20.4% in 2012. Similar upwards movements are observed for the poverty gap and the Watts index. In terms of inequality the Gini index remains stable at a level of around 0.280 in the 1970s. It then increases to 0.339 in 1990 followed by a further surge to a level of 0.371 at the turn of the century in 2000. In the last period of observation it has reached a value of 0.430.

In the following section we turn to the estimates of unfair inequality, i.e. inequality that can be traced back to violations of the normative ideals of EOp and FfP.

## 4.2 Results

**Baseline Results.** The gray bars in Figure 1 illustrate unfair inequality for the sample of 30 European countries. For these baseline estimates we use the MN-measure with  $r = 0$  to aggregate divergences between the empirical outcome distribution and the norm distribution **EOp+FfP**. The black crosses indicate total inequality as measured by the mean log deviation.

Clearly, the numeric values of the MN-measure are difficult to interpret. Therefore, we also convert our measure of unfair inequality into a percentage term as follows:

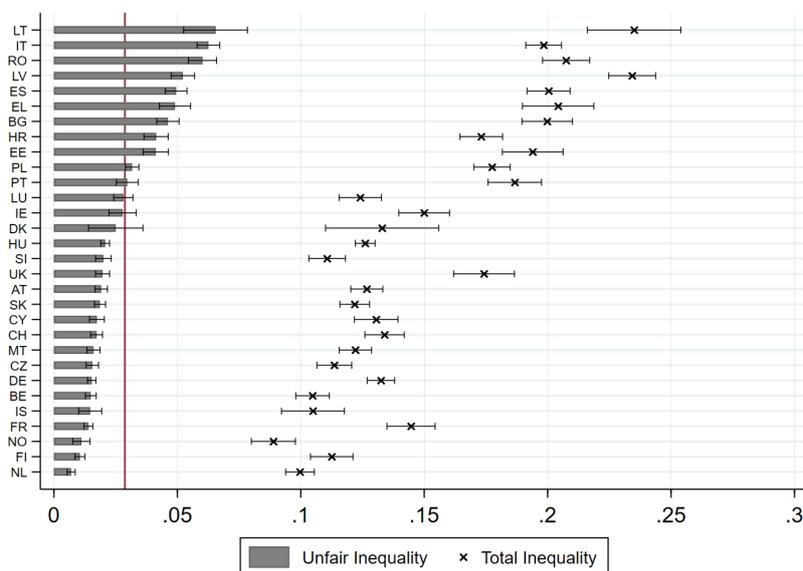
$$\frac{D(Y^e || Y_{\text{EOp+FfP}}^r)}{D(Y^e || \mu)}. \quad (5)$$

The numerator is our previously outlined measure of unfair inequality as informed by the principles of EOp and FfP. The denominator is a reference value of unfair inequality, in which all

observed inequality in  $Y^e$  is judged to be unfair since the norm value for all  $i \in N$  is the population mean  $\mu$ . In case of our baseline measure of divergences – the MN-measure with  $r = 0$  – this term collapses to the mean log deviation. Therefore, the relative measure is the ratio of the values indicated by the gray bars and the black crosses in Figure 1. It shows how much of total inequality remains of ethical concern once we specify norm incomes according to EOp and FfP. Clearly, the conversion into a relative measure is not innocuous. Consider two countries with equal levels of  $D(Y^e||Y_{\text{EOp+FfP}}^r)$ , but unequal levels of total inequality. Due to the division by  $D(Y^e||\mu)$  the relative measure would characterize the country with lower total inequality as more unfair than the country with higher total inequality. For example, with a value of 0.011 Norway ranks at the lower end of the cross-country distribution of unfair inequality. When dividing unfair inequality by total inequality, however, the country is propelled to the 6th position just because its total level of inequality is relatively low. In view of this misleading property, we refrain from cross-country and longitudinal comparisons based on the relative measure. However, in the following we will also report percentage values in brackets in order to facilitate the interpretation of our results.<sup>22</sup>

Unfair inequality is most prevalent in the countries of Lithuania, Italy and Romania, in which the MN-measure with  $r = 0$  attains values of 0.066 (27.9%), 0.063 (31.6%) and 0.060 (29.0%), respectively. From the perspective of our normative stance income is most fairly distributed in the Netherlands (0.007, 7.0%), Finland (0.011, 9.3%) and Norway (0.011, 12.5%). We find that on average unfair inequality amounts to 0.029, i.e. that 17.6% of total inequality can be explained by violations of EOp and FfP.

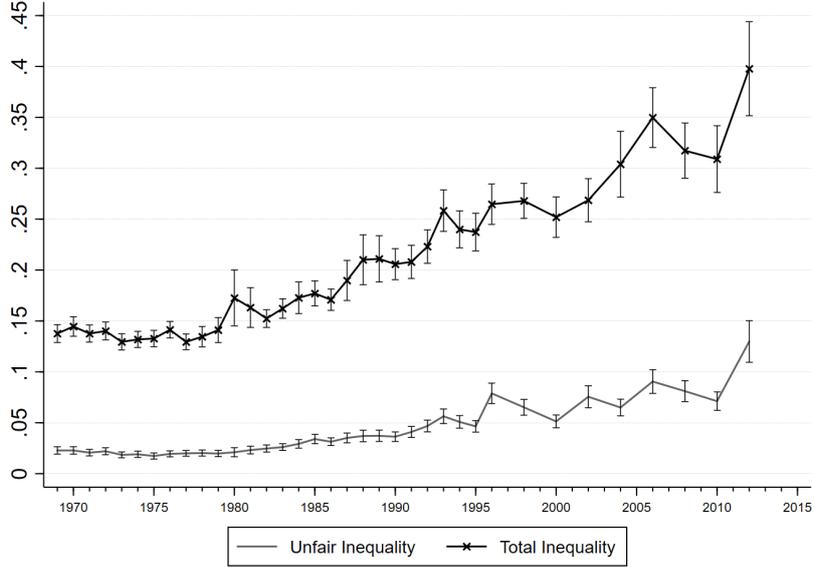
Figure 1: Unfair Inequality by Country (Europe)



**Data:** EU-SILC 2011 cross-sectional (rev.5, June 2015).  
**Note:** 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws. Unfair inequality is calculated as  $D(Y^e||Y^r)$ . This term measures the aggregate divergence between the empirical distribution  $Y^e$  and the norm distribution  $Y^r$ . Divergences are aggregated based on MN with  $r = 0$ . The norm distribution is given by  $Y_{\text{EOp+FfP}}^r$ . See also Table 1 for information on the calculation of norm distributions. The vertical red line indicates the unweighted cross-country average. Point estimates are presented in Table F.1.

<sup>22</sup>Overviews of point estimates for the relative measures are reported in Tables D.1 and D.2 in the Appendix.

Figure 2: Unfair Inequality over Time (USA)



**Data:** PSID public use dataset and PSID-CNEF.  
**Note:** 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws. Unfair inequality is calculated as  $D(Y^e || Y^r)$ . This term measures the aggregate divergence between the empirical distribution  $Y^e$  and the norm distribution  $Y^r$ . Divergences are aggregated based on MN with  $r = 0$ . The norm distribution is given by  $Y_{\text{EOP+FfP}}^r$ . See also Table 1 for information on the calculation of norm distributions. Point estimates are presented in Table F.2.

Figure 2 illustrates the evolution of unfair inequality in the US over time. The gray line yields our baseline measure of unfair inequality while the black crosses again are indicative of total inequality as measured by the mean log deviation.

Starting from a level of 0.023 (16.6%) in 1969, unfair inequality attained a level of 0.130 (32.6%) in 2012. The co-movement of total inequality and unfair inequality shows an interesting pattern. Up until 1980 both total inequality and unfair inequality were relatively stable. The 1980s and early 1990s were characterized by strong increases in inequality from 0.173 in 1980 up to 0.237 in 1995. These increases, however, were only weakly related to violations of either EOp or FfP. In this period only 39% of the well-documented rally in inequality levels (Piketty et al., 2018) must be ascribed to increasing unfairness. After 1995 total inequality further increased to 0.398 in 2012. In contrast to the previous period, however, 52% of the increase in inequality can be accounted for by increases in unfairness. Unfair inequality in the US exceeds the corresponding levels of the most unjust European societies, such as Italy and Lithuania, by far in the last periods of observation.

**Decomposition.** The previous discussion provokes the question to what extent our baseline results are driven by violations of one principle rather than the other. Therefore, in this section we decompose unfair inequality into its EOp and its FfP components. We cannot cleanly allocate all unfair inequality to either FfP or EOp, since they are partly overlapping. To illustrate this point, consider a society where all  $i \in P$  belong to the same type  $t_p \in T$ . By EOp we know that  $\mu_{t_p}^r = \mu$  in the norm distribution. Re-scaling all incomes in  $t_p$  to realize EOp thus also entails that a fraction of  $i \in t_p$  will be lifted out of poverty. In this case, by coincidence an opportunity egalitarian planner would also be considered poverty-averse. Reversely,  $i \in t_p$  is congruent with

$i \in P$ . Therefore, rescaling all incomes below the poverty line to realize FfP simultaneously lifts the incomes of  $t_p$  closer to the population mean. In this sense, aiming for FfP also entails opportunity equalization. Therefore, the upper bound share of unfairness explained by violations of either EOp or FfP is given by how much unfair inequality we can attribute to either principle once we abstract from the other. The construction of these norm distributions were discussed in section 2 and are referenced as scenarios **EOp Only** and **FfP Only** in Table 1. The respective lower bound of how much unfair inequality we can attribute to either principle is given by how much unfair inequality rises once we add this principle to the other. The decomposition of unfair inequality into the two components thus reads as follows:

$$D(Y^e || Y_{\text{EOp+FfP}}^r) = \underbrace{D(Y^e || Y_{\text{EOp+FfP}}^r) - D(Y^e || Y_{\text{EOp}}^r)}_{\text{LB}_{\text{FfP}}} + \underbrace{D(Y^e || Y_{\text{EOp}}^r)}_{\text{UB}_{\text{EOp}}}$$

$$D(Y^e || Y_{\text{EOp+FfP}}^r) = \underbrace{D(Y^e || Y_{\text{EOp+FfP}}^r) - D(Y^e || Y_{\text{FfP}}^r)}_{\text{LB}_{\text{EOp}}} + \underbrace{D(Y^e || Y_{\text{FfP}}^r)}_{\text{UB}_{\text{FfP}}}$$

As can be easily inferred from the above formulas, summing upper and lower bounds across principles yields the results presented in Figures 1 and 2. For the following comparisons we will refer to the upper bound measure of EOp and the lower bound measure of FfP, that is we allocate the entirety of the overlap component to EOp. This has the advantage that we can benchmark our results against standard measures of inequality of opportunity.<sup>23</sup>

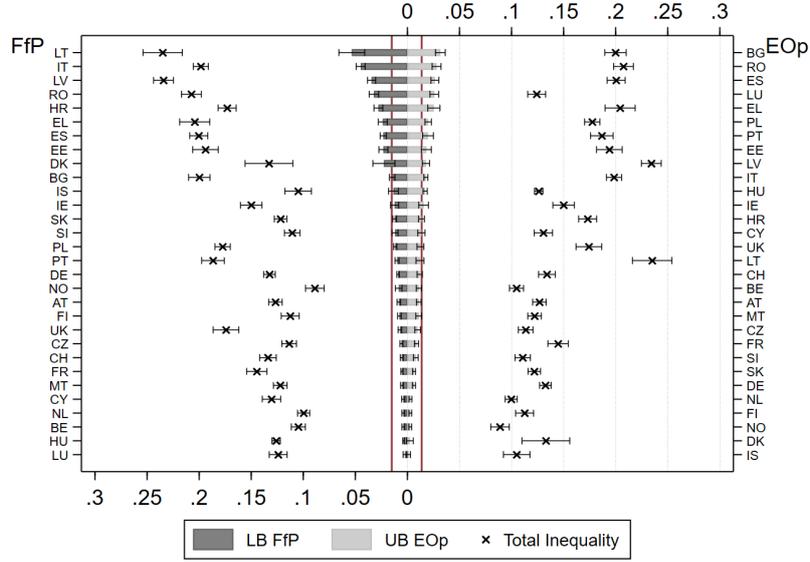
As documented in Figures 3 and 4 our baseline results are driven by EOp and FfP in about equal proportions. In the right-hand panel of Figure 3 we show upper-bound unfair inequality attributable to violations of EOp in Europe in 2010. In the left-hand panel we show the corresponding lower bounds for the extent of unfair inequality attributable to FfP. A standard opportunity egalitarian ( $\text{UB}_{\text{EOp}}$ ) would detect an average unfairness level of 0.014 (8.6%) in the income distributions of our sample. From her perspective the distributions of Bulgaria (0.032, 15.8%), Romania (0.028, 13.5%) and Spain (0.026, 13.2%) would raise most moral concern. On the other side of the spectrum we find the Nordic countries Iceland, Denmark and Norway, in all of which unfair inequality due to EOp is lower than 0.003.

Augmenting the concern for equal opportunities with poverty aversion ( $\text{LB}_{\text{FfP}}$ ), unfair inequality on average increases by 0.015 (9.0pp). The strongest increases are observed for Lithuania (0.053, 22.7pp), Italy (0.045, 22.6pp) and Latvia (0.034, 14.7pp). With increases of at most 0.04 points Luxembourg, Hungary and Belgium experience the lowest upwards correction of unfair inequality when complementing EOp with FfP. As outlined in section 3 our measure of unfair inequalities as originating from FfP is closely related to the Watts index. Therefore, it does not come as a surprise that the upward correction in unfair inequalities coming from FfP is strongly aligned with the country-specific Watts index. The Pearson's correlation coefficient between  $\text{LB}_{\text{FfP}}$  and the Watts index amounts to 0.946 and 0.965 for the EU-SILC and the PSID sample, respectively (Figure E.1).

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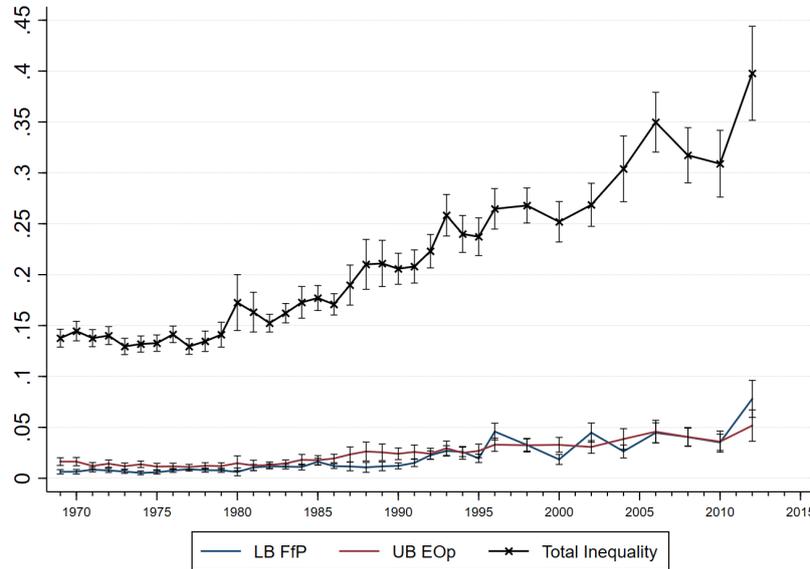
<sup>23</sup>Recall that  $D(Y^e || Y_{\text{EOp}}^r)$  collapses to the standard measure of EOp in which the smoothed distribution of type means is evaluated by the mean log deviation, see Table 1. A decomposition of unfair inequality into the respective lower bound components of EOp and FfP as well as the overlap component is shown in Tables D.1 and D.2.

Figure 3: Decomposition by Country (Europe)



**Data:** EU-SILC 2011 cross-sectional (rev.5, June 2015).  
**Note:** 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws. Unfair inequality is calculated as  $D(Y^e || Y^r)$ . This term measures the aggregate divergence between the empirical distribution  $Y^e$  and the norm distribution  $Y^r$ . Divergences are aggregated based on MN with  $\tau = 0$ . The right-hand panel shows the upper bound estimate of unfair inequality attributable to EOp. Here the norm distribution is given by  $Y_{EOp}^r$ . The left-hand panel shows the lower bound estimate of unfair inequality attributable to FfP. The lower bound measure is calculated as the difference in the divergence measures for the norms  $Y_{EOp+FfP}^r$  and  $Y_{EOp}^r$ . See also Table 1 for information on the calculation of norm distributions. The vertical red line indicates the unweighted cross-country average.

Figure 4: Decomposition over Time (USA)

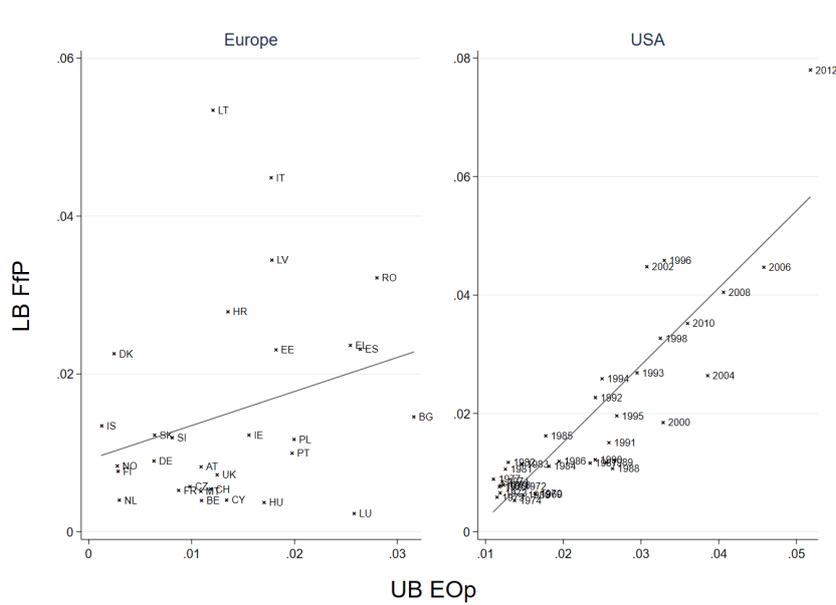


**Data:** PSID public use dataset and PSID-CNEF.  
**Note:** 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws. Unfair inequality is calculated as  $D(Y^e || Y^r)$ . This term measures the aggregate divergence between the empirical distribution  $Y^e$  and the norm distribution  $Y^r$ . Divergences are aggregated based on MN with  $\tau = 0$ . The red line shows the upper bound estimate of unfair inequality attributable to EOp. Here the norm distribution is given by  $Y_{EOp}^r$ . The blue line shows the lower bound estimate of unfair inequality attributable to FfP. The lower bound measure is calculated as the difference in the divergence measures for the norms  $Y_{EOp+FfP}^r$  and  $Y_{EOp}^r$ . See also Table 1 for information on the calculation of norm distributions.

In analogy to the cross-sectional pattern observed in Europe, the development of unfair inequality in the US is attributable to increasing violations of both EOp and FfP (Figure 4). The trends of the series only diverge in the late 1980s when the contribution of FfP slightly decreased and subsequently stagnated at its level until the mid 1990s. Otherwise, the contributions of both sources of unfairness are steadily increasing over the time period of observation. The significant contribution of FfP suggests that an exclusive focus on top income inequality (Piketty et al., 2018) may be misguided – especially when it comes to analyzing inequality patterns with an eye to fairness considerations.

Figure 5 shows the correlation pattern of  $UB_{EOp}$  and  $LB_{FfP}$ . With values of 0.276 (Europe) and 0.881 (US) the correlations between  $UB_{EOp}$  and  $LB_{FfP}$  are positive and sizable. This indicates that countries with strong opportunity differences are also characterized by more severe violations of FfP. Furthermore, the US example shows evidence of poverty increases over time being accompanied by rising inequality of opportunity.<sup>24</sup>

Figure 5: Correlation of EOp and FfP



**Data:** EU-SILC 2011 cross-sectional (rev.5, June 2015). PSID public use dataset and PSID-CNEF.  
**Note:** Correlations are based on the upper bound measure for EOp and the lower bound measure for FfP, respectively. Unfair inequality is calculated as  $D(Y^e || Y^r)$ . This term measures the aggregate divergence between the empirical distribution  $Y^e$  and the norm distribution  $Y^r$ . Divergences are aggregated based on MN with  $r = 0$ . See also Table 1 for information on the calculation of norm distributions. The Pearson's correlation coefficients are 0.276 and 0.881, respectively.

### 4.3 Sensitivity Analysis

We conduct several sensitivity checks including alternations in the specific divergence measure, alternative poverty thresholds and modifications of our normative requirements.

**Alternative Divergence Measures.** All results presented until this point were based on the divergence measures proposed by Magdalou and Nock (2011) with  $r = 0$ . In addition to

<sup>24</sup>Furthermore, current research suggests that contemporary increases in poverty may translate into future opportunity shortages for children of affected families (Heckman and Mosso, 2014; Hoynes et al., 2016). Hence, one would expect a positive correlation between current poverty and future inequality of opportunity as well.

alternations in the weighting parameter  $r$ , we present results based on the measures put forward by Cowell (1985) and Almás et al. (2011) in Tables F.1 and F.2.

Within the MN-family, an increase of  $r$  induces a shift in the weight put on FfP and EOp, respectively. With increasing  $r$  the measure puts less weight on negative divergences of  $Y^e$  from  $Y^r$ . Since  $\forall i \in P, y_i^e \leq y_i^r = y_{\min}$  this translates into decreasing relative importance of FfP. To the contrary, increasing  $r$  puts more weight on EOp since this principle induces stronger positive divergences between  $Y^e$  from  $Y^r$  in the middle and upper parts of the distribution. Just as the measures proposed by Magdalou and Nock (2011), the family put forward by Cowell (1985) is a generalization of the general entropy class of inequality measures and thus varies with the parameter specification of  $r$ .<sup>25</sup> Therefore, within the Cowell-class the same observations with respect to the parameter  $r$  apply. The Cowell-measure and the MN-family coincide exactly for  $r = 1$ . Lastly, we employ the unfairness Gini proposed by Almás et al. (2011),<sup>26</sup> which is a generalization of the well-known Gini index. The unfairness Gini tends to put relatively less weight on large negative divergences from the reference distribution, which results in decreases of the importance of FfP as opposed to EOp as explanatory factor for the unfair share of inequality.

Table 2: Rank Correlation of Measures by Country (Europe)

	Magdalou and Nock			Cowell		Almás et al.
	r = 0 (Baseline)	r = 1	r = 2	r = 1	r = 2	r = 3
<b>Magdalou and Nock</b>						
r=0	1.000					
r=1	0.957	1.000				
r=2	0.916	0.981	1.000			
<b>Cowell</b>						
r=0	0.978	0.988	0.962	1.000		
r=1	0.957	1.000	0.981	0.988	1.000	
r=2	0.945	0.994	0.985	0.976	0.994	1.000
<b>Almás</b>						
	0.919	0.970	0.970	0.956	0.970	0.968
						1.000

**Data:** EU-SILC 2011 cross-sectional (rev.5, June 2015).

**Note:** The coefficients indicate rank correlation coefficients between the point estimates of unfair inequality as calculated by the different divergence measures. Unfair inequality is calculated as  $D(Y^e||Y^r)$ . This term measures the aggregate divergence between the empirical distribution  $Y^e$  and the norm distribution  $Y^r$ . The norm distribution is given by  $Y_{\text{EOP+FfP}}^r$ . See also Table 1 for information on the calculation of norm distributions.

In spite of these variations, the measures yield highly comparable results in terms of cross-country and -time comparisons. Tables 2 and 3 show rank-correlations for the different measures and their parameterizations for the European and the US sample, respectively. All correlation

<sup>25</sup>To be precise the Cowell-family aggregates divergences as follows:

$$D(Y^e||Y^r) = \sum_i \left[ y_i^r \phi \left( \frac{y_i^e}{y_i^r} \right) \right],$$

where

$$\phi(z) = \begin{cases} -\ln z, & \text{if } r = 0 \\ z \ln z, & \text{if } r = 1 \\ \frac{1}{r(r-1)}z, & \text{otherwise.} \end{cases}$$

<sup>26</sup>To be precise the unfairness Gini aggregates divergences as follows:

$$D(Y^e||Y^r) = \frac{2}{N(N-1)\mu} \sum_i i(y_i^r - y_i^e).$$

coefficients lie above the level of 0.910. Hence, we conclude that our results are robust to alternations in the way in which divergences between  $Y^e$  and  $Y^r$  are aggregated.

Table 3: Rank Correlation of Measures over Time (USA)

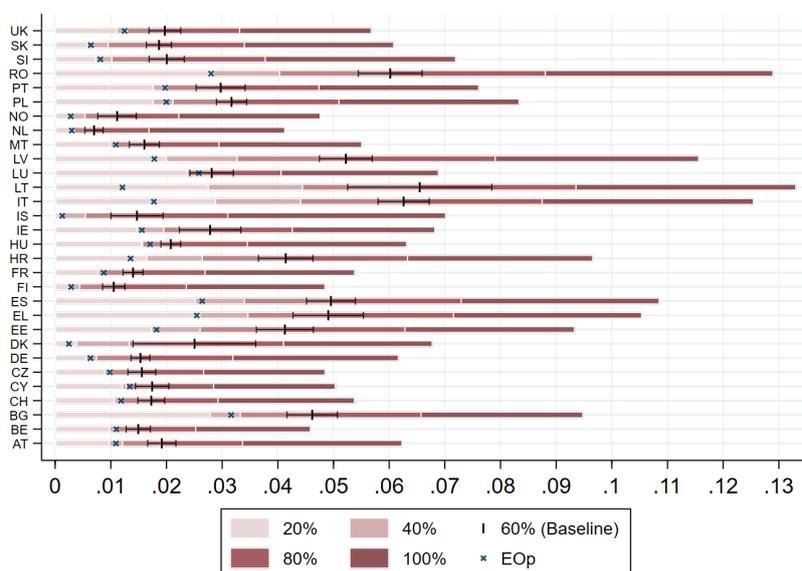
	Magdalou and Nock			Cowell		Almås et al.
	r = 0 (Baseline)	r = 1	r = 2	r = 1	r = 2	r = 3
<b>Magdalou and Nock</b>						
r=0	1.000					
r=1	0.991	1.000				
r=2	0.961	0.975	1.000			
<b>Cowell</b>						
r=0	0.994	0.998	0.971	1.000		
r=1	0.991	1.000	0.975	0.998	1.000	
r=2	0.986	0.998	0.979	0.994	0.998	1.000
<b>Almås</b>						
	0.972	0.984	0.966	0.978	0.984	0.985

**Data:** PSID public use dataset and PSID-CNEF.

**Note:** The coefficients indicate rank correlation coefficients between the point estimates of unfair inequality as calculated by the different divergence measures. Unfair inequality is calculated as  $D(Y^e||Y^r)$ . This term measures the aggregate divergence between the empirical distribution  $Y^e$  and the norm distribution  $Y^r$ . The norm distribution is given by  $Y_{EOP+FFP}^r$ . See also Table 1 for information on the calculation of norm distributions.

**Alternative Poverty Thresholds.** Naturally, what it means “to make ends meet” is contentious both philosophically (e.g. Casal, 2007) and empirically (e.g. Meyer and Sullivan, 2012). It is beyond the ambit of this work to resolve these issues. However, in Figures 6 and 7 we illustrate the sensitivity of our proposed measure to variations in the level of  $y_{\min}$ .

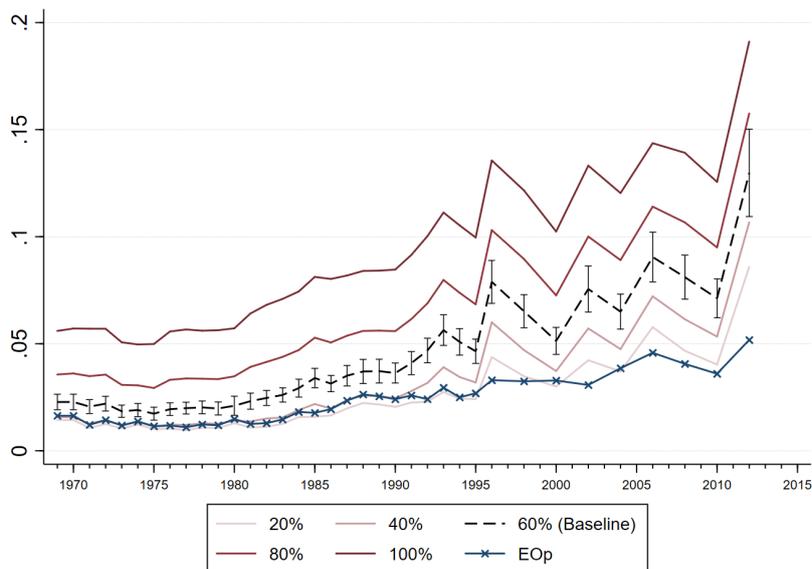
Figure 6: Alternative Poverty Thresholds by Country (Europe)



**Data:** EU-SILC 2011 cross-sectional (rev.5, June 2015).

**Note:** 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws. Except for the baseline estimate, confidence bands are not shown for reasons of visual clarity. Unfair inequality is calculated as  $D(Y^e||Y^r)$ . This term measures the aggregate divergence between the empirical distribution  $Y^e$  and the norm distribution  $Y^r$ . Divergences are aggregated based on MN with  $r = 0$ . The norm distribution is given by  $Y_{EOP+FFP}^r$ . See also Table 1 for information on the calculation of norm distributions. Alternative poverty thresholds are calculated as percentage of the country-specific equivalized disposable household median income. The black dashes indicate our baseline estimates. The blue crosses indicate the upper bound measure of unfair inequality attributable to EOP.

Figure 7: Alternative Poverty Thresholds over Time (USA)



**Data:** PSID public use dataset and PSID-CNEF.

**Note:** 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws. Except for the baseline estimate, confidence bands are not shown for reasons of visual clarity. Unfair inequality is calculated as  $D(Y^e || Y^r)$ . This term measures the aggregate divergence between the empirical distribution  $Y^e$  and the norm distribution  $Y^r$ . Divergences are aggregated based on MN with  $r = 0$ . The norm distribution is given by  $Y_{EOP+FfP}^r$ . See also Table 1 for information on the calculation of norm distributions. Alternative poverty thresholds are calculated as percentage of the year-specific equivalized disposable household median income. The black dashed line indicates our baseline estimates. The blue crosses indicate the upper bound measure of unfair inequality attributable to EOp.

In particular, we vary the percentage level of the population median income that is required to trespass the poverty threshold in 20 percentage point steps. By definition at the 100%-mark  $y_{\min}$  would equal the population's median income and 50% of the population would be in  $P$  and  $R$ , respectively. If we set  $y_{\min} = 0$ , we would obtain the upper bound measure of EOp. The 60%-mark replicates the baseline results presented in Figures 1 and 2. Intuitively, the unfair share of inequality varies positively with the deprivation threshold: The higher  $y_{\min}$ , the stronger our inequality aversion with respect to inequalities within circumstance types.<sup>27</sup> The susceptibility of our measure of unfair inequality to alternations of the deprivation threshold is remarkably country-specific. For example, in countries in which the lowest percentiles of the income distribution are strong determinants of observed total inequality, even the lowest deprivation threshold (20% of the median) yields strong upward corrections of unfair inequality in comparison to the standard EOp measure. Cases in point are the examples of Lithuania and Italy. To the contrary, in countries in which inequality is a phenomenon of increasing income dispersion in the upper percentiles, introducing FfP with a very stringent poverty threshold has little effect on the unfair share of inequality. Instead, for FfP to provide substantial upward corrections in comparison to standard EOp measures one would have to rely on a rather lenient definition of what it means to be deprived. An examples for this latter group of countries is

<sup>27</sup>Note that there are a some exceptions for very low poverty thresholds. For example, in the Netherlands the standard EOp measure yields higher unfair inequality than the combined measure up until the 20%-poverty mark. Similarly, in the US the standard EOp measure consistently exceeds the combined measure with at a poverty threshold of 20% up until 1995. Intuitively, low levels of poverty require very little re-scaling of incomes below the poverty threshold for FfP to be satisfied. As a consequence, inversely re-scaling incomes by type means to achieve EOp may overshoot the requirements of FfP, especially if differences in type-means are rather pronounced.

furnished by Luxembourg in which even the upper bound EOp measure (0% poverty threshold) lies within the 95% confidence band of our baseline measure (60% poverty threshold).

The specification of the poverty threshold is a strong driver of our empirical results. All alternative poverty scenario lie outside the 95% confidence bands of our baseline measure. The same holds for all observations in the US sample.

**Alternative Norm Distributions.** In a last step, we investigate three variations in the way we reconcile FfP and EOp by varying the set of eligible norm distributions as given by the intersection  $\cap_{s=1}^5 D^s$  in section 3.

**Alternative (a).** In our baseline scenario, we constructed the norm-vector by first redistributing from  $R$  to  $P$  to realize FfP. In a second step we redistributed among the non-poor of each type in order to realize EOp. Recall that opportunity equalization was achieved by a linear tax rate on the incomes exceeding  $y_{\min}$  in order to leave FfP intact. Thus,  $\forall i \in P$  we conceded dominance to the principle of FfP over EOp. To illustrate this, consider the incomes of  $i, j \in t$  such that  $y_j^e = y_i^e - \epsilon = y_{\min} < y_i^e$  where  $\epsilon$  is an arbitrarily small positive quantity. Thus,  $i$  and  $j$  are from the same circumstance type and are close to indistinguishable in terms of their effort exertion  $\theta \in \Theta$ . In our baseline scenario the corresponding norm incomes would be  $y_j^r = y_{\min}$  whereas  $y_i^r = y_i^e \left[ 1 - \left( \frac{y_i^e - y_{\min}}{y_i^e} \right) (\tau^{\text{FfP}} + \tau_t^{\text{EOp}} (1 - \tau^{\text{FfP}})) \right]$ . Since we pin down the norm income of those below the poverty line to equal  $y_{\min}$ , their income cannot exceed the poverty line irrespective of whether they come from a very disadvantaged or an advantaged type. If in our example,  $\tau_t^{\text{EOp}}$  was negative – as it should for disadvantaged types with  $\mu_t < \mu$  – individual  $i$  would benefit from opportunity equalization, while  $j$  would not. Thus, around the poverty threshold there is a discontinuity in the extent to which the members of one circumstance type benefit from opportunity equalization.

To circumvent this property one has to concede dominance to the principle of EOp over the principle of FfP in the lower end of the distribution. Thus, we define the sets  $P^{(a)}$  and  $R^{(a)}$  such that  $P^{(a)} = \{i : y_i^e (1 - \frac{\mu_t - \mu}{\mu_t}) \leq y_{\min}\}$  and  $R^{(a)} = \{i : y_i^e (1 - \frac{\mu_t - \mu}{\mu_t}) > y_{\min}\}$ . Note that in contrast to our baseline scenario we now identify the deprived after their incomes had been re-scaled to realize EOp. While **Constant Resources** and **EOp** are unaffected by this re-definition, it entails changes in the remaining three set restrictions:

$$D^{3(a)} = \left\{ D : y_i^r = y_{\min} \quad \forall i \in P^{(a)} \right\}. \quad (\text{FfP (a)})$$

$$D^{4(a)} = \left\{ D : y_i^r \geq y_{\min} \quad \forall i \in R^{(a)} \right\}. \quad (\text{Financing I (a)})$$

$$D^{5(a)} = \left\{ D : \forall t \in T, \frac{y_i^r - y_{\min}}{y_j^r - y_{\min}} = \frac{y_i^e - y_{\min}}{y_j^e - y_{\min}} \quad \forall i, j \in t \cap R^{(a)} \right\}. \quad (\text{Financing II (a)})$$

The intersection  $\cap_{s=1}^5 D^{s(a)}$  again yields a singleton:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e (1 - \tau_t^{\text{EOp}(a)}) \leq y_{\min} \\ y_i^e \left[ 1 - \tau_t^{\text{EOp}(a)} - \tilde{y}_i^{(a)} \tau_t^{\text{FfP}(a)} (1 - \tau_t^{\text{EOp}(a)}) \right], & \text{otherwise.} \end{cases} \quad (\text{Alternative (a)})$$

where  $\tilde{y}_i^{(a)} = \left( \frac{y_i^e (1 - \tau_t^{\text{EOp}(a)}) - y_{\min}}{y_i^e (1 - \tau_t^{\text{EOp}(a)})} \right)$ ,  $\tau_t^{\text{FfP}(a)} = \frac{N_{P^{(a)} \cap t} \left[ \frac{y_{\min} - \mu_{P^{(a)} \cap t}^e (1 - \tau_t^{\text{EOp}(a)})}{\mu_{P^{(a)} \cap t}^e} \right]}{N_{R^{(a)} \cap t} \left[ \frac{\mu_{R^{(a)} \cap t}^e (1 - \tau_t^{\text{EOp}(a)}) - y_{\min}}{\mu_{R^{(a)} \cap t}^e} \right]}$  and  $\tau_t^{\text{EOp}(a)} = \frac{\mu_t - \mu}{\mu_t}$ .

Verbally the construction the alternative norm vector can be described as follows: We first redistribute across types by means of linear transfers to realize **EOp**. In contrast to our baseline scenario, all members of a type including households below the deprivation threshold are eligible for the respective transfers. Furthermore, the tax base for these transfers is all income without invoking the poverty line. We then conduct bottom fill-up to satisfy **FfP (a)**. By **Financing I (a)** and **Financing II (a)** the necessary funds are raised by linear tax rates that are applied on the share of income exceeding the poverty threshold. The tax rate must be type-specific not to upset **EOp**, which requires equal mean incomes across types. In this sense, poverty eradication follows from Pigou-Dalton transfers within types that have no influence on inequality between types.

While such a norm distribution eliminates the discontinuity around the poverty threshold, it imposes another discontinuity across circumstance types. Since tax rates for poverty eradication are now type-specific, the norm income of high income persons in  $R^{(a)}$ , decreases with the type-specific poverty gap ratio: The more funds are necessary to eradicate intra-type poverty, the higher the linear tax-rate on the non-deprived type members. To illustrate this, consider the incomes of  $i \in t_m$  and  $j \in t_n$  such that  $y_j^e = y_i^e$  and  $\mu_m^e = \mu_n^e$ . Thus,  $i$  and  $j$  are from different circumstance types that, however, are on par in terms of the value of their respective opportunity sets. In our baseline scenario the corresponding norm incomes would be  $y_j^r = y_i^r = y_i^e \left[ 1 - \left( \frac{y_i^e - y_{\min}}{y_i^e} \right) (\tau^{\text{FFP}} + \tau_t^{\text{EOP}} (1 - \tau^{\text{FFP}})) \right]$ . Both households would receive the same norm income as they have the same background characteristics ( $\mu_m^e = \mu_n^e$ ) to transform resources into income and share the same responsibility to alleviate deprivation in society ( $\tau_m^{\text{FFP}} = \tau_n^{\text{FFP}} = \tau^{\text{FFP}} \forall t \in T$ ). To the contrary, in the alternative scenario as specified by the intersection  $\cap_{s=1}^5 D^{s(a)}$ ,  $\tau_t^{\text{FFP(a)}}$  is not constant across types but varies with the average shortfall (surplus) of incomes in  $P^{(a)} \cap t$  ( $R^{(a)} \cap t$ ) from the poverty line  $y_{\min}$ . The greater the divergence between  $\tau_m^{\text{FFP}}$  and  $\tau_n^{\text{FFP}}$  the greater the divergence between the norm incomes of  $y_i^r$  and  $y_j^r$ . Thus, instead of interpreting the eradication of poverty as a shared burden across types, the alternative scenario suggests the eradication of poverty to be a type-specific endeavor.

In spite of their different normative implications, the empirical differences between the baseline and this alternative scenario are negligible. In both the European cross-section (Figure 8, Panel 1) and the longitudinal US sample (Figure 9, Panel 1), the alternative scenario yields slightly higher values of unfair inequality. However, in every country-year cell, the point estimate of the alternative specification lies comfortably within the 95% confidence band of our baseline estimate.

**Alternative (b).** In our baseline scenario, we assume that the value of type-specific opportunity sets should be evaluated by reference to all  $i \in t$ . However, one may also claim that the requirement of opportunity equalization only operates for households whose basic needs are satisfied. In line with this normative assumption we replace **EOp** by the following requirement:

$$D^{2(b)} = \left\{ D : \mu_{t \cap R}^r = \frac{1}{N_{t \cap R}} \sum_{i \in t \cap R} y_i^r = \frac{1}{N_R} \sum_{i \in R} y_i^r = \mu_R \forall t \in T \right\}. \quad (\text{EOP (b)})$$

Instead of rating type-specific advantages by the type mean, **EOP (b)** draws on the average excess income above the poverty line to evaluate opportunity sets. Hence, according to this

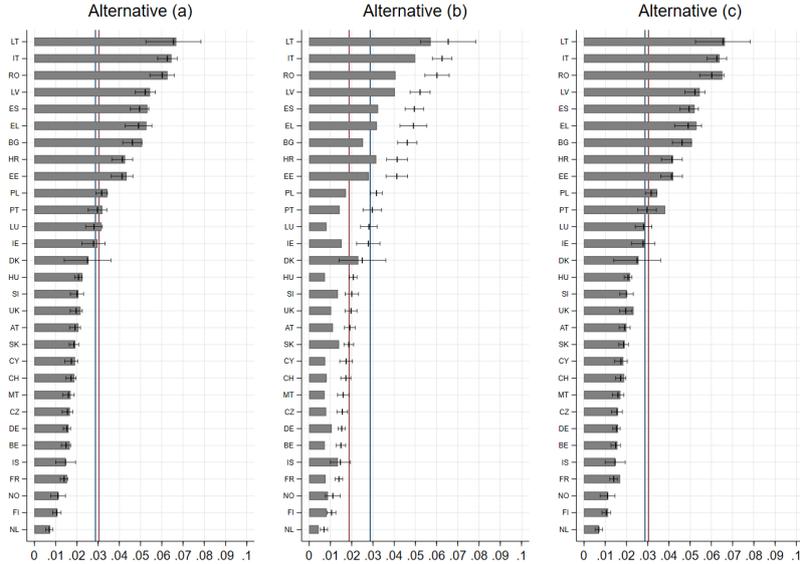
alternative interpretation the norm distribution would be constructed as follows:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^r \leq y_{\min} \\ y_i^e [1 - \tilde{y}_i (\tau^{\text{FFP}} + \tau_t^{\text{EOP}^{(b)}}(1 - \tau^{\text{FFP}}))], & \text{otherwise.} \end{cases} \quad (\text{Alternative (b)})$$

where  $\tilde{y}_i = \left( \frac{y_i^e - y_{\min}}{y_i^e} \right)$ ,  $\tau^{\text{FFP}} = \frac{N_P(y_{\min} - \mu_P^e)}{N_R(\mu_R^e - y_{\min})}$  and  $\tau_t^{\text{EOP}^{(b)}} = \frac{\mu_{R \cap t}^e - \tau^{\text{FFP}} \left( \frac{N_{R \cap t}}{N_t} (\mu_{R \cap t}^e - y_{\min}) \right) - \mu_R^e}{\mu_{R \cap t}^e - \tau^{\text{FFP}} \left( \frac{N_{R \cap t}}{N_t} (\mu_{R \cap t}^e - y_{\min}) \right) - y_{\min}}$ .

In comparison to our baseline measure this alternative interpretation leads to decreases in the extent of inequality that is deemed unfair. Restricting the evaluation of opportunity sets to incomes above the poverty line, makes the distribution of type-specific advantages more homogeneous and therefore requires less transfers across types. As a consequence,  $D(Y^e || Y_{\text{EOP+FFP}}^r)$  on average decreases by approximately one third in both the European (Figure 8, Panel 2) and the US sample (Figure 9, Panel 2).

Figure 8: Alternative Norm Distributions by Country (Europe)

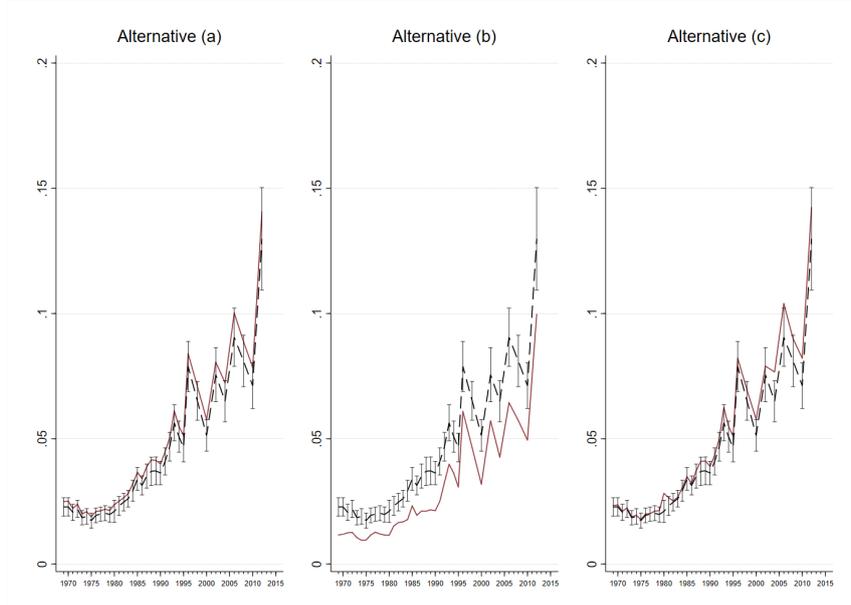


**Data:** EU-SILC 2011 cross-sectional (rev.5, June 2015).

**Note:** 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws. Except for the baseline estimate, confidence bands are not shown for reasons of visual clarity. Unfair inequality is calculated as  $D(Y^e || Y^r)$ . This term measures the aggregate divergence between the empirical distribution  $Y^e$  and the norm distribution  $Y^r$ . Divergences are aggregated based on MN with  $r = 0$ . The gray bars show unfair inequality calculated by reference to the respective alternative scenario indicated in the panel header. The black dashes indicate our baseline estimates. The vertical blue and red lines indicate the unweighted cross-country average for the baseline and the alternative estimate, respectively.

**Alternative (c).** Note that up to this point we maintained zero inequality aversion within circumstance types conditional on the fact that all  $i \in P$  were lifted to the minimal income  $y_{\min}$ . However, there may be many arguments for cushioning income disparity beyond our concerns for EOP and FfP. Therefore, in analogy to the poverty threshold  $y_{\min}$  we introduce an affluence threshold  $y_{\max}$  and consider any income exceeding this threshold as unfair. Normatively such a requirement may be defended by reference to concerns about bargaining power and political capture such as in Piketty et al. (2014). As a consequence, we partition  $R = \{i : y_i^e > y_{\min}\}$  into a middle class group,  $M^{(c)} = \{i : y_{\min} < y_i^e < y_{\max}\}$  and an affluent group,  $A^{(c)} = \{i : y_i^e \geq y_{\max}\}$

Figure 9: Alternative Norm Distributions over Time (USA)



**Data:** PSID public use dataset and PSID-CNEF.  
**Note:** 95% confidence intervals are calculated based on a bootstrap procedure with 500 draws. Except for the baseline estimate, confidence bands are not shown for reasons of visual clarity. Unfair inequality is calculated as  $D(Y^e || Y^r)$ . This term measures the aggregate divergence between the empirical distribution  $Y^e$  and the norm distribution  $Y^r$ . Divergences are aggregated based on MN with  $r = 0$ . The red lines show unfair inequality calculated by reference to the respective alternative scenario indicated in the panel header. The black dashed line indicates our baseline estimates.

in order to adjust **Financing II** as follows:

$$D^{5(c)} = \left\{ D : \forall t \in T, \frac{y_i^r - y_{\min}}{y_j^r - y_{\min}} = \frac{y_i^e - y_{\min}}{y_j^e - y_{\min}} \quad \forall i, j \in t \cap M^{(c)} \right\}. \quad (\text{Financing II (c)})$$

**Financing II (c)** reflects the fact that zero inequality aversion with respect to incomes exceeding the poverty threshold only exists for the non-affluent middle class  $M$ . On all incomes that exceed the affluence threshold we impose a freedom from affluence (FfA) restriction:

$$D^{6(c)} = \left\{ D : y_i^r = y_{\max} \quad \forall i \in A^{(c)} \right\}. \quad (\text{FfA})$$

By **FfA** there is infinite inequality aversion for the share of income exceeding the affluence threshold. While maintaining all other restrictions as in the baseline scenario, the norm distribution is now pinned down to a singleton by  $\cap_{s=1}^6 D^{s(c)}$ :

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^r \leq y_{\min} \\ y_{\max}, & \text{if } y_i^r \geq y_{\max} \\ y_i^e [1 - \tilde{y}_i (\tau^{\text{FFP}} + \tau_t^{\text{EOP}} (1 - \tau^{\text{FFP}}))], & \text{otherwise.} \end{cases} \quad (\text{Alternative (c)})$$

where  $\tilde{y}_i = \left( \frac{y_i^e - y_{\min}}{y_i^e} \right)$ ,  $\tau_t^{\text{EOP}} = \frac{\mu_t^e + \frac{N_P \cap t}{N_t} (y_{\min} - \mu_{P \cap t}^e) - \tau^{\text{FFP}(c)} \left( \frac{N_{M^{(c)} \cap t}}{N_t} (\mu_{M^{(c)} \cap t}^e - y_{\min}) \right) - \frac{N_{A^{(c)} \cap t}}{N_t} (\mu_{A^{(c)} \cap t}^e - y_{\max}) - \mu}{\mu_t^e + \frac{N_P \cap t}{N_t} (y_{\min} - \mu_{P \cap t}^e) - \tau^{\text{FFP}(c)} \left( \frac{N_{M^{(c)} \cap t}}{N_t} (\mu_{M^{(c)} \cap t}^e - y_{\min}) \right) - \frac{N_{A^{(c)} \cap t}}{N_t} (\mu_{A^{(c)} \cap t}^e - y_{\max}) - y_{\min}}$   
and  $\tau^{\text{FFP}(c)} = \frac{N_P (y_{\min} - \mu_P^e) - N_{A^{(c)}} (\mu_{A^{(c)}}^e - y_{\max})}{N_{M^{(c)}} (\mu_{M^{(c)}}^e - y_{\min})}$ .

Just as in the case of poverty lines, the appropriate setting of affluence thresholds is a

contentious and normatively laden issue (Peichl et al., 2010; Atkinson and Brandolini, 2013). For this empirical exercise we set the affluence line to 400% of the country- and year-specific median income. The ensuing results for Europe are shown in the right-hand panels of Figures 8 and 9. Complementing our baseline principles – EOp and FfP – with FfA leads to little changes in the extent of unfair inequality in both sample. As expected, the introduction of FfA leads to upward adjustments in the extent of unfair inequality. However, all estimates lie comfortably within the 95% confidence band of our baseline estimate. This finding holds although the affluence threshold arguably is very low. On the one hand, it is hardly defensible to apply a marginal tax rate of 100% once a household disposes of four times the resources available to the median household. On the other hand, increasing the affluence threshold would lead to decreases in the extent of inequality aversion within circumstance types and thus lower levels of unfair inequality. Therefore, these results re-emphasize our previous claim that an exclusive focus on high income earners may be misleading – especially when it comes to fairness evaluations of income distribution.

## 5 Conclusion

In this paper we have provided a new measure of unfair inequality that reconciles the ideals of equal opportunities and absence of poverty. In fact we provide the first work that combines the measurement of equality of opportunity and poverty by treating both as co-equal grounds for compensation. As such, we differ from previous works that have analyzed both principles in a joint framework by prioritizing either FfP or EOp.

While standard measures of EOp can be interpreted as information on the extent of unfair inequalities in a given outcome distribution, concerns have been expressed about them producing very low estimates of unfair inequality due to the partial observability of circumstances (Kanbur and Wagstaff, 2016). In this paper we propose a new route to address the lower bound nature of current measures of unfair inequality by effectively acknowledging that judgments on the fairness of an outcome distribution are informed by multiple normative principles.

Empirical application of our measures in a standardized dataset of 30 European countries and a longitudinal sample for the US show important implications for current discourse on inequality. First, we indeed find sizable upward corrections of unfair inequality when complementing the concern for equal opportunities with poverty aversion. While the extent of this upward correction remains sensitive to the exact specification of the deprivation threshold, our results are largely robust to the use of different inequality indexes and slight modifications of our normative requirements. Second, we show that our unfairness measures are much more sensitive to variations at the bottom of the income distribution than at the top. This suggests to re-shift the recent focus of attention from the upmost parts of the income distribution to the lower percentiles if the issue at stake is unfairness instead of a mere description of aggregate inequality.

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## A Construction of Norm Vector

**Freedom form Poverty.** **FfP** requires bottom fill-up so that  $y_i^r = y_{\min} \forall i \in P$ . Invoking **Constant Resources** the funds for bottom fill up must be raised in group  $R$ , where by **Financing I** the tax base is given by  $y_i^e - y_{\min}$ . Again invoking **Constant Resources** it must be the case that  $\sum_P (y_{\min} - y_i^e) = \sum_R \tau_i^{\text{FfP}} (y_i^e - y_{\min})$ . Abstracting from **EOP**, there is zero inequality aversion for the share of income exceeding  $y_{\min}$  (**Financing II**) and therefore  $\tau_i^{\text{FfP}} = \tau_j^{\text{FfP}} = \tau^{\text{FfP}} \forall i, j \in R$ . Since  $\tau_i^{\text{FfP}}$  must be linear,  $\tau^{\text{FfP}} = \frac{N_P(y_{\min} - \mu_P^e)}{N_R(\mu_R^e - y_{\min})}$ . Hence, the only distribution complying with **Constant Resources**, **FfP**, **Financing I** and **Financing II** is given by:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e \leq y_{\min} \\ y_i^e [1 - \tilde{y}_i(\tau^{\text{FfP}})], & \text{otherwise.} \end{cases}, \quad (\text{FfP Only})$$

where  $\tilde{y}_i = \left( \frac{y_i^e - y_{\min}}{y_i^e} \right)$  and  $\tau^{\text{FfP}} = \frac{N_P(y_{\min} - \mu_P^e)}{N_R(\mu_R^e - y_{\min})}$ .

**Equality of Opportunity.** Let's consider **FfP Only** as an intermediate distribution that has to be reshaped by means of the tax rate  $\tau_i^{\text{EOp}}$  in order to comply with **EOP**. By **FfP** we know that  $\tau_i^{\text{EOp}} = 0 \forall i \in P$ . Invoking **Financing I** it must be the case that the tax base for  $\tau_i^{\text{EOp}}$  is  $y_i^e [1 - \tilde{y}_i(\tau^{\text{FfP}})] - y_{\min} \forall i \in R$ . By **Financing II**,  $\tau_i^{\text{EOp}} = \tau_j^{\text{EOp}} = \tau_t^{\text{EOp}} \forall i, j \in R \cap t, \forall t \in T$ . Taken together these provisions specify, that **EOP** must be realized by means of transfers among the rich constituents of each type. These transfers are linear within types and calculated on the share of income exceeding the poverty threshold. Therefore:

$$y_i^r = \begin{cases} y_{\min}, & \text{if } y_i^e \leq y_{\min} \\ \{y_i^e [1 - \tilde{y}_i(\tau^{\text{FfP}})] - y_{\min}\}(1 - \tau_t^{\text{EOp}}) + y_i^e [1 - \tilde{y}_i(\tau^{\text{FfP}} + \tau_t^{\text{EOp}}(1 - \tau^{\text{FfP}}))], & \text{otherwise.} \end{cases}$$

To realize **EOP** the following must hold:  $\mu_t^r = \frac{1}{N_t} \sum_{i \in t} y_i^r = \frac{1}{N_t} \sum_{i \in t} y_i^e = \mu, \forall t \in T$ . Hence, it must be the case that:

$$\frac{1}{N_t} \left\{ \sum_{i \in t \cap P} y_{\min} + \sum_{i \in t \cap R} y_i^e [1 - \tilde{y}_i(\tau^{\text{FfP}} + \tau_t^{\text{EOp}}(1 - \tau^{\text{FfP}}))] \right\} = \mu, \forall t \in T.$$

$$\text{Solving for } \tau_t^{\text{EOp}} \text{ then yields: } \tau_t^{\text{EOp}} = \frac{\left[ \mu_t^e + \frac{N_{P \cap t}}{N_t} (y_{\min} - \mu_{P \cap t}^e) - \tau^{\text{FfP}} \left( \frac{N_{R \cap t}}{N_t} (\mu_{R \cap t}^e - y_{\min}) \right) \right] - \mu}{\left[ \mu_t^e + \frac{N_{P \cap t}}{N_t} (y_{\min} - \mu_{P \cap t}^e) - \tau^{\text{FfP}} \left( \frac{N_{R \cap t}}{N_t} (\mu_{R \cap t}^e - y_{\min}) \right) \right] - y_{\min}}.$$

## B Descriptive Statistics

Table B.1: Descriptive Statistics by Country (Europe)

	N	$\mu$	P <sub>0</sub>	P <sub>1</sub>	Watts Index	Gini	MLD	Types
AT	6,350	25,590 (195)	0.131 (0.006)	0.032 (0.002)	0.043 (0.003)	0.270 (0.003)	0.127 (0.003)	36
BE	5,407	24,116 (182)	0.105 (0.005)	0.024 (0.001)	0.031 (0.002)	0.244 (0.004)	0.105 (0.003)	35
BG	6,931	3,798 (38)	0.163 (0.005)	0.056 (0.002)	0.081 (0.004)	0.332 (0.004)	0.200 (0.005)	30
CH	6,897	42,240 (350)	0.106 (0.005)	0.026 (0.002)	0.034 (0.002)	0.279 (0.004)	0.134 (0.004)	36
CY	4,906	21,142 (219)	0.104 (0.005)	0.025 (0.002)	0.033 (0.002)	0.278 (0.005)	0.131 (0.004)	36
CZ	6,752	9,038 (73)	0.098 (0.005)	0.025 (0.002)	0.033 (0.002)	0.256 (0.004)	0.114 (0.004)	29
DE	12,316	22,394 (133)	0.146 (0.004)	0.037 (0.001)	0.047 (0.002)	0.276 (0.003)	0.132 (0.003)	36
DK	2,532	30,803 (483)	0.111 (0.012)	0.039 (0.006)	0.062 (0.012)	0.258 (0.009)	0.133 (0.012)	32
EE	5,374	7,178 (90)	0.162 (0.006)	0.057 (0.003)	0.085 (0.005)	0.324 (0.005)	0.194 (0.006)	36
EL	6,331	13,458 (166)	0.187 (0.006)	0.060 (0.003)	0.091 (0.005)	0.331 (0.006)	0.204 (0.007)	35
ES	15,360	17,359 (130)	0.188 (0.005)	0.063 (0.002)	0.095 (0.004)	0.329 (0.003)	0.200 (0.004)	36
FI	4,563	25,958 (240)	0.120 (0.006)	0.027 (0.002)	0.035 (0.003)	0.253 (0.005)	0.113 (0.004)	36
FR	11,145	24,572 (197)	0.124 (0.004)	0.027 (0.001)	0.035 (0.002)	0.290 (0.005)	0.145 (0.005)	36
HR	5,947	6,722 (59)	0.167 (0.005)	0.056 (0.002)	0.087 (0.004)	0.302 (0.003)	0.173 (0.004)	36
HU	13,583	5,394 (30)	0.137 (0.003)	0.029 (0.001)	0.037 (0.001)	0.276 (0.002)	0.126 (0.002)	31
IE	3,069	25,386 (349)	0.136 (0.008)	0.034 (0.003)	0.049 (0.005)	0.292 (0.005)	0.150 (0.005)	36
IS	1,579	20,616 (291)	0.122 (0.011)	0.035 (0.004)	0.048 (0.006)	0.241 (0.007)	0.105 (0.007)	34
IT	20,152	18,985 (98)	0.184 (0.004)	0.066 (0.002)	0.115 (0.004)	0.310 (0.002)	0.198 (0.004)	36
LT	5,295	4,810 (62)	0.193 (0.008)	0.073 (0.004)	0.129 (0.010)	0.340 (0.005)	0.235 (0.010)	34
LU	6,871	38,243 (400)	0.140 (0.006)	0.029 (0.002)	0.037 (0.003)	0.272 (0.005)	0.124 (0.004)	36
LV	6,437	5,457 (52)	0.195 (0.005)	0.071 (0.002)	0.111 (0.004)	0.353 (0.004)	0.234 (0.005)	36
MT	4,255	13,408 (128)	0.130 (0.006)	0.028 (0.002)	0.036 (0.002)	0.269 (0.004)	0.122 (0.003)	36
NL	5,513	24,007 (213)	0.092 (0.007)	0.017 (0.002)	0.022 (0.002)	0.245 (0.003)	0.100 (0.003)	36
NO	2,493	40,730 (416)	0.092 (0.008)	0.023 (0.003)	0.032 (0.004)	0.221 (0.005)	0.089 (0.005)	36
PL	14,616	6,233 (44)	0.168 (0.003)	0.047 (0.001)	0.064 (0.002)	0.320 (0.003)	0.177 (0.004)	23
PT	5,923	11,036 (128)	0.153 (0.005)	0.042 (0.002)	0.057 (0.003)	0.332 (0.005)	0.187 (0.006)	33
RO	7,565	2,575 (24)	0.211 (0.006)	0.076 (0.003)	0.116 (0.005)	0.332 (0.004)	0.207 (0.005)	23
SI	4,870	13,124 (115)	0.143 (0.008)	0.040 (0.003)	0.054 (0.004)	0.248 (0.004)	0.111 (0.004)	36
SK	7,288	7,494 (47)	0.127 (0.005)	0.036 (0.002)	0.050 (0.003)	0.259 (0.003)	0.122 (0.003)	32
UK	6,242	23,320 (263)	0.123 (0.005)	0.031 (0.002)	0.041 (0.002)	0.319 (0.006)	0.174 (0.006)	36

**Data:** EU-SILC 2011 cross-sectional (rev.5, June 2015).

**Note:** Standard errors are calculated based on a bootstrap procedure with 500 draws and reported in parentheses.

Table B.2: Descriptive Statistics over Time (USA)

	N	$\mu$	$P_0$	$P_1$	Watts Index	Gini	MLD	Types
1969	2,552	4,839 (59)	0.116 (0.007)	0.034 (0.002)	0.045 (0.003)	0.278 (0.005)	0.138 (0.004)	36
1970	2,638	5,224 (67)	0.119 (0.007)	0.035 (0.002)	0.047 (0.003)	0.285 (0.005)	0.145 (0.005)	36
1971	2,758	5,561 (63)	0.121 (0.006)	0.037 (0.002)	0.049 (0.003)	0.277 (0.004)	0.138 (0.004)	36
1972	2,868	6,043 (71)	0.121 (0.006)	0.035 (0.002)	0.048 (0.003)	0.280 (0.005)	0.140 (0.004)	36
1973	2,980	6,567 (73)	0.112 (0.006)	0.032 (0.002)	0.042 (0.003)	0.270 (0.004)	0.129 (0.004)	36
1974	3,071	7,174 (83)	0.108 (0.006)	0.029 (0.002)	0.038 (0.003)	0.274 (0.004)	0.132 (0.004)	36
1975	3,242	7,656 (81)	0.106 (0.006)	0.029 (0.002)	0.039 (0.003)	0.277 (0.004)	0.133 (0.004)	36
1976	3,316	8,501 (95)	0.122 (0.006)	0.034 (0.002)	0.045 (0.003)	0.284 (0.004)	0.141 (0.004)	36
1977	3,509	8,837 (85)	0.123 (0.006)	0.035 (0.002)	0.048 (0.003)	0.269 (0.004)	0.129 (0.004)	36
1978	3,607	9,619 (109)	0.117 (0.006)	0.034 (0.002)	0.046 (0.003)	0.275 (0.005)	0.135 (0.005)	36
1979	3,764	10,440 (136)	0.121 (0.006)	0.034 (0.002)	0.045 (0.003)	0.282 (0.006)	0.141 (0.006)	36
1980	3,835	11,593 (224)	0.129 (0.006)	0.035 (0.002)	0.046 (0.003)	0.308 (0.011)	0.173 (0.014)	36
1981	3,955	12,173 (196)	0.133 (0.006)	0.041 (0.002)	0.056 (0.003)	0.300 (0.009)	0.163 (0.010)	36
1982	4,054	12,800 (137)	0.147 (0.006)	0.042 (0.002)	0.058 (0.004)	0.292 (0.004)	0.152 (0.004)	36
1983	4,119	13,822 (150)	0.149 (0.006)	0.046 (0.002)	0.062 (0.003)	0.301 (0.005)	0.162 (0.005)	36
1984	4,248	14,845 (207)	0.153 (0.006)	0.046 (0.002)	0.064 (0.003)	0.310 (0.007)	0.173 (0.008)	36
1985	4,306	15,568 (187)	0.152 (0.006)	0.051 (0.002)	0.074 (0.004)	0.310 (0.006)	0.177 (0.006)	36
1986	4,371	16,130 (187)	0.157 (0.006)	0.051 (0.002)	0.070 (0.003)	0.309 (0.005)	0.171 (0.005)	36
1987	4,456	18,049 (285)	0.155 (0.006)	0.051 (0.002)	0.072 (0.004)	0.323 (0.008)	0.190 (0.010)	36
1988	4,497	19,352 (331)	0.163 (0.006)	0.052 (0.002)	0.073 (0.004)	0.342 (0.010)	0.210 (0.013)	36
1989	4,549	20,320 (351)	0.157 (0.006)	0.052 (0.002)	0.074 (0.004)	0.342 (0.009)	0.211 (0.012)	36
1990	4,593	20,925 (294)	0.168 (0.006)	0.053 (0.002)	0.074 (0.003)	0.339 (0.007)	0.206 (0.008)	36
1991	4,659	21,365 (309)	0.169 (0.006)	0.056 (0.002)	0.081 (0.004)	0.339 (0.007)	0.208 (0.008)	36
1992	4,553	22,586 (329)	0.172 (0.007)	0.060 (0.003)	0.093 (0.005)	0.346 (0.007)	0.223 (0.008)	36
1993	4,624	22,618 (353)	0.175 (0.007)	0.067 (0.003)	0.106 (0.006)	0.369 (0.008)	0.258 (0.010)	36
1994	4,896	23,118 (331)	0.177 (0.006)	0.064 (0.003)	0.100 (0.005)	0.357 (0.007)	0.240 (0.009)	36
1995	4,898	24,422 (366)	0.175 (0.006)	0.060 (0.003)	0.091 (0.004)	0.360 (0.007)	0.237 (0.009)	36
1996	3,808	25,936 (394)	0.178 (0.007)	0.071 (0.003)	0.131 (0.007)	0.359 (0.007)	0.265 (0.010)	36
1998	4,052	27,774 (416)	0.184 (0.006)	0.071 (0.003)	0.117 (0.006)	0.373 (0.006)	0.268 (0.009)	36
2000	4,027	32,204 (571)	0.173 (0.007)	0.061 (0.003)	0.093 (0.005)	0.371 (0.008)	0.252 (0.010)	36
2002	4,042	31,870 (489)	0.186 (0.007)	0.070 (0.003)	0.127 (0.008)	0.365 (0.007)	0.269 (0.011)	36
2004	4,948	36,390 (810)	0.193 (0.007)	0.072 (0.003)	0.114 (0.005)	0.402 (0.011)	0.304 (0.016)	36
2006	4,861	39,197 (805)	0.185 (0.006)	0.073 (0.003)	0.137 (0.008)	0.419 (0.009)	0.350 (0.015)	36
2008	4,763	40,007 (814)	0.201 (0.007)	0.077 (0.004)	0.134 (0.008)	0.403 (0.009)	0.317 (0.014)	36

Continued on next page

Table B.2 – *Continued from previous page*

	N	$\mu$	P <sub>0</sub>	P <sub>1</sub>	Watts Index	Gini	MLD	Types
2010	4,595	39,732 (883)	0.183 (0.007)	0.071 (0.003)	0.120 (0.006)	0.399 (0.011)	0.309 (0.017)	36
2012	4,406	42,461 (1,142)	0.204 (0.007)	0.088 (0.004)	0.187 (0.013)	0.430 (0.013)	0.398 (0.024)	36

**Data:** PSID public use dataset and PSID-CNEF.

**Note:** Standard errors are calculated based on a bootstrap procedure with 500 draws and reported in parentheses.

## C Occupational Classification

Table C.1: Occupational Classification

	Census 1970	Census 1990	ISCO-08
High	(1) Professional, Technical and Kindred workers	(1) Managerial and Professional Specialty Occ.	(1) Managers
	(2) Managers, Officials and Proprietors	(2) Technical and Sales Op.	(2) Professionals
	(3) Self-Employed Businessmen	–	(3) Technicians and Associate Professionals
Middle	(4) Clerical and Sales Workers	(3) Administrative Support Occ., Including Clerical	(4) Clerical Support Workers
	(5) Craftsmen, Foremen and Kindred Workers	(5) Precision Production, Craft, and Repair Occ.	(7) Craft and Related Trade Workers
	(6) Operatives and Kindred Workers	(7) Machine Op., Assemblers, and Inspectors	(8) Plant and Machine Op.s and Assemblers
	–	(6) Extractive and Precision Production Occ.	–
Low	(7) Laborers, Service Workers and Farm Laborers	(4) Service, Farming, Forestry, and Fishing Occ.	(6) Skilled Agric., Forestry and Fishery Workers
	(8) Farmers and Farm Managers	(8) Transportation and Material Moving Occ., Handlers, Equipment Cleaners, Helpers, and Laborers	(9) Elementary Occ.
	(9) Miscellaneous (incl. Armed Services, Protective Workers etc.)	(9) Military Occ.	(0) Armed Forces Occ.
	(-) Not in Labor Force	(-) Not in Labor Force	(-) Not in Labor Force

**Note:** In the PSID, waves 1970-2001 report occupation codes with reference to 1970 Census codes. Waves 2003-2013 report occupation codes with reference to 2000 Census codes. If available on 3-digit level we use the cross-walk routine provided by Autor and Dorn (2013) to standardize codes based on the 1990 Census classification. Few occupational codes available in the PSID are not included in the routine provided by Autor and Dorn (2013). These groups are allocated to their 1990 Census classification analogues by the authors of this paper. We then aggregate all codes to the 1-digit level and apply the classification scheme outlined above. In EU-SILC, the 2005 wave reports occupation codes with reference to the ISCO-88 classification, while the 2011 wave reports occupation codes with reference to the ISCO-08 classification. We apply the cross-walk routine provided by the International Labour Organization to standardize codes based on the ISCO-08 classification. We then aggregate all codes to the 1-digit level and apply the classification scheme outlined above.

## D Upper and Lower Bounds of FfP and EOp

Table D.1: Relative Point Estimates by Country (Europe)

	Baseline			Decomposition		
	N	$\mu$	$D(Y^e    Y_{\text{EOp+FfP}}^r)$	$\text{LB}_{\text{EOp}}$	$\text{LB}_{\text{FfP}}$	Overlap
AT	6,350	25,590	15.1%	6.3%	6.5%	2.3%
BE	5,407	24,116	14.2%	7.3%	3.8%	3.1%
BG	6,931	3,798	23.1%	10.4%	7.3%	5.4%
CH	6,897	42,240	12.9%	6.8%	4.1%	2.1%
CY	4,906	21,142	13.3%	7.6%	3.1%	2.6%
CZ	6,752	9,038	13.7%	6.7%	5.1%	2.0%
DE	12,316	22,394	11.6%	3.5%	6.8%	1.2%
DK	2,532	30,803	18.8%	1.4%	17.0%	0.4%
EE	5,374	7,178	21.3%	6.8%	11.9%	2.6%
EL	6,331	13,458	24.0%	8.4%	11.6%	4.0%
ES	15,360	17,359	24.7%	8.6%	11.6%	4.6%
FI	4,563	25,958	9.3%	2.1%	6.8%	0.5%
FR	11,145	24,572	9.7%	4.3%	3.6%	1.7%
HR	5,947	6,722	23.9%	5.7%	16.1%	2.1%
HU	13,583	5,394	16.5%	10.6%	3.0%	2.9%
IE	3,069	25,386	18.6%	8.4%	8.2%	2.0%
IS	1,579	20,616	14.0%	1.1%	12.8%	0.1%
IT	20,152	18,985	31.6%	6.4%	22.6%	2.5%
LT	5,295	4,810	27.9%	3.5%	22.7%	1.6%
LU	6,871	38,243	22.7%	16.1%	1.9%	4.7%
LV	6,437	5,457	22.3%	5.1%	14.7%	2.5%
MT	4,255	13,408	13.1%	7.2%	4.2%	1.7%
NL	5,513	24,007	7.0%	2.5%	4.0%	0.4%
NO	2,493	40,730	12.5%	2.6%	9.4%	0.5%
PL	14,616	6,233	17.8%	8.1%	6.6%	3.1%
PT	5,923	11,036	15.9%	8.3%	5.3%	2.3%
RO	7,565	2,575	29.0%	9.4%	15.5%	4.0%
SI	4,870	13,124	18.1%	5.9%	10.8%	1.4%
SK	7,288	7,494	15.3%	3.8%	10.1%	1.5%
UK	6,242	23,320	11.3%	5.4%	4.2%	1.8%

**Data:** EU-SILC 2011 cross-sectional (rev.5, June 2015).

**Note:** Unfair inequality is calculated as  $D(Y^e || Y^r)$ . This term measures the aggregate divergence between the empirical distribution  $Y^e$  and the norm distribution  $Y^r$ . Divergences are aggregated based on MN with  $\tau = 0$ . Rows titled  $D(Y^e || Y_{\text{EOp+FfP}}^r)$  indicate the baseline measure of total unfair inequality attributable to both EOp and FfP. Rows titled  $\text{LB}_{\text{EOp}}$  indicate the lower bound measure of unfair inequality attributable to EOp. Rows titled  $\text{LB}_{\text{FfP}}$  indicate the lower bound measure of unfair inequality attributable to FfP. Rows titled Overlap indicate the overlap component that can be attributed to either EOp or FfP. See also Table 1 for information on the calculation of norm distributions.

Table D.2: Relative Point Estimates over Time (USA)

	Baseline			Decomposition		
	N	$\mu$	$D(Y^e    Y_{EOP+FFP}^r)$	LB <sub>EOP</sub>	LB <sub>FFP</sub>	Overlap
1969	2,552	4,839	16.6%	8.1%	4.7%	3.8%
1970	2,638	5,224	15.8%	7.6%	4.4%	3.8%
1971	2,758	5,561	15.0%	5.9%	6.2%	2.9%
1972	2,868	6,043	15.7%	6.7%	5.5%	3.5%
1973	2,980	6,567	14.3%	6.1%	5.1%	3.0%
1974	3,071	7,174	14.4%	7.2%	4.0%	3.2%
1975	3,242	7,656	13.1%	5.9%	4.4%	2.8%
1976	3,316	8,501	13.7%	5.6%	5.4%	2.8%
1977	3,509	8,837	15.4%	5.6%	6.9%	2.9%
1978	3,607	9,619	15.1%	6.1%	5.9%	3.0%
1979	3,764	10,440	14.0%	5.8%	5.5%	2.6%
1980	3,835	11,593	12.2%	5.5%	3.6%	3.0%
1981	3,955	12,173	14.2%	4.9%	6.5%	2.8%
1982	4,054	12,800	16.2%	5.3%	7.7%	3.1%
1983	4,119	13,822	16.1%	5.7%	7.1%	3.3%
1984	4,248	14,845	16.9%	6.7%	6.4%	3.8%
1985	4,306	15,568	19.2%	6.1%	9.2%	3.9%
1986	4,371	16,130	18.4%	7.0%	7.0%	4.4%
1987	4,456	18,049	18.5%	7.4%	6.1%	5.0%
1988	4,497	19,352	17.6%	7.6%	5.1%	4.9%
1989	4,549	20,320	17.6%	7.4%	5.6%	4.7%
1990	4,593	20,925	17.6%	7.3%	5.9%	4.4%
1991	4,659	21,365	19.7%	7.6%	7.3%	4.8%
1992	4,553	22,586	21.0%	6.4%	10.2%	4.4%
1993	4,624	22,618	21.8%	6.4%	10.4%	5.0%
1994	4,896	23,118	21.2%	6.1%	10.8%	4.3%
1995	4,898	24,422	19.6%	6.7%	8.3%	4.7%
1996	3,808	25,936	29.8%	6.8%	17.3%	5.7%
1998	4,052	27,774	24.3%	7.0%	12.2%	5.1%
2000	4,027	32,204	20.4%	7.7%	7.3%	5.3%
2002	4,042	31,870	28.1%	6.9%	16.7%	4.6%
2004	4,948	36,390	21.4%	7.4%	8.7%	5.3%
2006	4,861	39,197	25.9%	7.5%	12.8%	5.6%
2008	4,763	40,007	25.6%	7.5%	12.8%	5.3%
2010	4,595	39,732	23.0%	7.0%	11.4%	4.6%
2012	4,406	42,461	32.6%	7.6%	19.6%	5.4%

**Data:** PSID public use dataset and PSID-CNEF.

**Note:** Unfair inequality is calculated as  $D(Y^e || Y^r)$ . This term measures the aggregate divergence between the empirical distribution  $Y^e$  and the norm distribution  $Y^r$ . Divergences are aggregated based on MN with  $r = 0$ . Rows titled  $D(Y^e || Y_{EOP+FFP}^r)$  indicate the baseline measure of total unfair inequality attributable to both EOp and FfP. Rows titled LB<sub>EOP</sub> indicate the lower bound measure of unfair inequality attributable to EOp. Rows titled LB<sub>FFP</sub> indicate the lower bound measure of unfair inequality attributable to FfP. Rows titled Overlap indicate the overlap component that can be attributed to either EOp or FfP. See also Table 1 for information on the calculation of norm distributions.



## F Divergence Measures

Table F.1: Divergence Measures by Country (Europe)

	Magdalou and Nock			Cowell			Almås et al.
	r = 0 (Baseline)	r = 1	r = 2	r = 0	r = 1	r = 2	
AT	0.019 (0.001)	0.016 (0.001)	0.024 (0.003)	0.018 (0.002)	0.016 (0.001)	0.015 (0.001)	0.098 (0.005)
BE	0.015 (0.001)	0.014 (0.001)	0.023 (0.004)	0.016 (0.002)	0.014 (0.001)	0.013 (0.001)	0.090 (0.005)
BG	0.046 (0.002)	0.041 (0.003)	0.072 (0.007)	0.043 (0.003)	0.041 (0.003)	0.041 (0.003)	0.172 (0.006)
CH	0.017 (0.001)	0.016 (0.001)	0.026 (0.004)	0.017 (0.002)	0.016 (0.001)	0.015 (0.002)	0.103 (0.005)
CY	0.017 (0.002)	0.017 (0.002)	0.028 (0.006)	0.017 (0.002)	0.017 (0.002)	0.018 (0.002)	0.098 (0.006)
CZ	0.016 (0.001)	0.014 (0.002)	0.021 (0.005)	0.014 (0.002)	0.014 (0.002)	0.014 (0.002)	0.097 (0.006)
DE	0.015 (0.001)	0.011 (0.001)	0.015 (0.002)	0.012 (0.001)	0.011 (0.001)	0.010 (0.001)	0.083 (0.004)
DK	0.025 (0.006)	0.011 (0.003)	0.012 (0.009)	0.015 (0.003)	0.011 (0.003)	0.009 (0.003)	0.059 (0.008)
EE	0.041 (0.003)	0.028 (0.003)	0.037 (0.005)	0.032 (0.003)	0.028 (0.003)	0.027 (0.003)	0.134 (0.008)
EL	0.049 (0.003)	0.038 (0.004)	0.061 (0.010)	0.041 (0.003)	0.038 (0.004)	0.038 (0.005)	0.141 (0.008)
ES	0.050 (0.002)	0.038 (0.002)	0.060 (0.007)	0.044 (0.002)	0.038 (0.002)	0.035 (0.002)	0.148 (0.005)
FI	0.011 (0.001)	0.007 (0.001)	0.007 (0.002)	0.007 (0.001)	0.007 (0.001)	0.006 (0.001)	0.056 (0.005)
FR	0.014 (0.001)	0.013 (0.001)	0.026 (0.004)	0.014 (0.001)	0.013 (0.001)	0.013 (0.001)	0.089 (0.005)
HR	0.041 (0.003)	0.024 (0.002)	0.027 (0.003)	0.029 (0.002)	0.024 (0.002)	0.023 (0.002)	0.116 (0.005)
HU	0.021 (0.001)	0.022 (0.001)	0.033 (0.002)	0.022 (0.001)	0.022 (0.001)	0.023 (0.001)	0.124 (0.004)
IE	0.028 (0.003)	0.022 (0.003)	0.030 (0.004)	0.024 (0.003)	0.022 (0.003)	0.022 (0.003)	0.121 (0.008)
IS	0.015 (0.002)	0.007 (0.001)	0.006 (0.002)	0.009 (0.002)	0.007 (0.001)	0.006 (0.001)	0.049 (0.007)
IT	0.063 (0.002)	0.034 (0.001)	0.041 (0.003)	0.044 (0.002)	0.034 (0.001)	0.031 (0.001)	0.130 (0.003)
LT	0.065 (0.007)	0.028 (0.003)	0.029 (0.004)	0.039 (0.004)	0.028 (0.003)	0.025 (0.003)	0.115 (0.007)
LU	0.028 (0.002)	0.032 (0.003)	0.053 (0.005)	0.035 (0.003)	0.032 (0.003)	0.031 (0.003)	0.152 (0.006)
LV	0.052 (0.002)	0.031 (0.002)	0.039 (0.004)	0.036 (0.002)	0.031 (0.002)	0.029 (0.002)	0.135 (0.006)
MT	0.016 (0.001)	0.015 (0.002)	0.021 (0.003)	0.015 (0.002)	0.015 (0.002)	0.016 (0.002)	0.093 (0.006)
NL	0.007 (0.001)	0.005 (0.001)	0.007 (0.001)	0.006 (0.001)	0.005 (0.001)	0.005 (0.001)	0.055 (0.005)
NO	0.011 (0.002)	0.007 (0.001)	0.006 (0.001)	0.008 (0.001)	0.007 (0.001)	0.006 (0.001)	0.056 (0.006)
PL	0.032 (0.001)	0.029 (0.002)	0.049 (0.006)	0.030 (0.002)	0.029 (0.002)	0.030 (0.002)	0.136 (0.005)
PT	0.030 (0.002)	0.029 (0.004)	0.049 (0.008)	0.027 (0.003)	0.029 (0.004)	0.034 (0.005)	0.116 (0.008)
RO	0.060 (0.003)	0.043 (0.003)	0.056 (0.005)	0.046 (0.002)	0.043 (0.003)	0.045 (0.003)	0.154 (0.006)
SI	0.020 (0.002)	0.015 (0.001)	0.017 (0.002)	0.016 (0.001)	0.015 (0.001)	0.014 (0.001)	0.091 (0.005)
SK	0.019 (0.001)	0.012 (0.001)	0.012 (0.001)	0.013 (0.001)	0.012 (0.001)	0.011 (0.001)	0.077 (0.004)
UK	0.020 (0.001)	0.017 (0.002)	0.030 (0.006)	0.018 (0.002)	0.017 (0.002)	0.017 (0.002)	0.109 (0.007)

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Table F.1 – *Continued from previous page*

Magdalou and Nock				Cowell		Almås et al.
r = 0 (Baseline)	r = 1	r = 2	r = 0	r = 1	r = 2	

**Data:** EU-SILC 2011 cross-sectional (rev.5, June 2015).

**Note:** Unfair inequality is calculated as  $D(Y^e || Y^r)$ . This term measures the aggregate divergence between the empirical distribution  $Y^e$  and the norm distribution  $Y^r$ . The norm distribution is given by  $Y_{\text{EOp+FP}}^r$ . See also Table 1 for information on the calculation of norm distributions. Divergences are aggregated according to the divergence measure indicated in the column header.

Table F.2: Divergence Measures over Time (USA)

	Magdalou and Nock			Cowell			Almås et al.
	r = 0 (Baseline)	r = 1	r = 2	r = 0	r = 1	r = 2	
1969	0.023 (0.002)	0.020 (0.002)	0.026 (0.004)	0.022 (0.002)	0.020 (0.002)	0.018 (0.002)	0.106 (0.007)
1970	0.023 (0.002)	0.020 (0.002)	0.027 (0.004)	0.022 (0.002)	0.020 (0.002)	0.019 (0.002)	0.109 (0.007)
1971	0.021 (0.002)	0.016 (0.002)	0.020 (0.003)	0.018 (0.002)	0.016 (0.002)	0.015 (0.002)	0.096 (0.007)
1972	0.022 (0.002)	0.018 (0.002)	0.024 (0.003)	0.020 (0.002)	0.018 (0.002)	0.017 (0.002)	0.104 (0.007)
1973	0.018 (0.001)	0.015 (0.002)	0.020 (0.003)	0.017 (0.002)	0.015 (0.002)	0.014 (0.002)	0.094 (0.007)
1974	0.019 (0.002)	0.017 (0.002)	0.023 (0.003)	0.018 (0.002)	0.017 (0.002)	0.016 (0.002)	0.100 (0.007)
1975	0.017 (0.002)	0.014 (0.002)	0.020 (0.003)	0.016 (0.002)	0.014 (0.002)	0.014 (0.002)	0.094 (0.007)
1976	0.019 (0.001)	0.015 (0.002)	0.020 (0.003)	0.017 (0.002)	0.015 (0.002)	0.014 (0.002)	0.092 (0.007)
1977	0.020 (0.001)	0.015 (0.001)	0.017 (0.002)	0.017 (0.002)	0.015 (0.001)	0.013 (0.001)	0.088 (0.005)
1978	0.020 (0.002)	0.016 (0.002)	0.021 (0.003)	0.018 (0.002)	0.016 (0.002)	0.014 (0.002)	0.093 (0.007)
1979	0.020 (0.002)	0.016 (0.002)	0.022 (0.005)	0.018 (0.002)	0.016 (0.002)	0.014 (0.002)	0.093 (0.007)
1980	0.021 (0.002)	0.019 (0.005)	0.053 (0.037)	0.021 (0.004)	0.019 (0.005)	0.018 (0.008)	0.095 (0.011)
1981	0.023 (0.002)	0.018 (0.003)	0.031 (0.019)	0.020 (0.003)	0.018 (0.003)	0.016 (0.004)	0.095 (0.009)
1982	0.025 (0.002)	0.018 (0.002)	0.023 (0.003)	0.021 (0.002)	0.018 (0.002)	0.016 (0.002)	0.101 (0.006)
1983	0.026 (0.002)	0.020 (0.002)	0.027 (0.004)	0.023 (0.002)	0.020 (0.002)	0.018 (0.002)	0.107 (0.007)
1984	0.029 (0.002)	0.024 (0.003)	0.041 (0.015)	0.027 (0.003)	0.024 (0.003)	0.022 (0.004)	0.123 (0.009)
1985	0.034 (0.002)	0.024 (0.003)	0.036 (0.007)	0.029 (0.003)	0.024 (0.003)	0.022 (0.003)	0.120 (0.008)
1986	0.031 (0.002)	0.025 (0.002)	0.037 (0.005)	0.029 (0.003)	0.025 (0.002)	0.023 (0.002)	0.124 (0.007)
1987	0.035 (0.002)	0.030 (0.005)	0.064 (0.034)	0.034 (0.004)	0.030 (0.005)	0.028 (0.006)	0.135 (0.010)
1988	0.037 (0.003)	0.033 (0.006)	0.081 (0.046)	0.037 (0.005)	0.033 (0.006)	0.031 (0.008)	0.148 (0.012)
1989	0.037 (0.003)	0.032 (0.005)	0.070 (0.033)	0.036 (0.005)	0.032 (0.005)	0.031 (0.007)	0.147 (0.011)
1990	0.036 (0.002)	0.031 (0.003)	0.052 (0.009)	0.034 (0.003)	0.031 (0.003)	0.029 (0.004)	0.144 (0.009)
1991	0.041 (0.003)	0.033 (0.004)	0.061 (0.015)	0.037 (0.004)	0.033 (0.004)	0.032 (0.005)	0.147 (0.009)
1992	0.047 (0.003)	0.032 (0.003)	0.049 (0.009)	0.039 (0.003)	0.032 (0.003)	0.029 (0.003)	0.141 (0.008)
1993	0.056 (0.004)	0.039 (0.004)	0.069 (0.015)	0.046 (0.004)	0.039 (0.004)	0.036 (0.004)	0.158 (0.010)
1994	0.051 (0.003)	0.035 (0.003)	0.062 (0.013)	0.041 (0.004)	0.035 (0.003)	0.032 (0.004)	0.150 (0.009)
1995	0.046 (0.003)	0.036 (0.004)	0.070 (0.016)	0.041 (0.004)	0.036 (0.004)	0.033 (0.004)	0.152 (0.010)
1996	0.079 (0.005)	0.044 (0.004)	0.070 (0.010)	0.059 (0.004)	0.044 (0.004)	0.039 (0.004)	0.162 (0.008)
1998	0.065 (0.004)	0.043 (0.004)	0.073 (0.010)	0.053 (0.004)	0.043 (0.004)	0.039 (0.004)	0.171 (0.009)
2000	0.051 (0.003)	0.042 (0.004)	0.091 (0.015)	0.048 (0.004)	0.042 (0.004)	0.039 (0.005)	0.174 (0.010)
2002	0.076 (0.005)	0.043 (0.004)	0.071 (0.013)	0.057 (0.004)	0.043 (0.004)	0.038 (0.004)	0.170 (0.008)
2004	0.065 (0.004)	0.049 (0.006)	0.123 (0.046)	0.058 (0.006)	0.049 (0.006)	0.045 (0.006)	0.191 (0.013)
2006	0.091 (0.006)	0.058 (0.006)	0.137 (0.029)	0.073 (0.006)	0.058 (0.006)	0.053 (0.008)	0.203 (0.012)
2008	0.081 (0.005)	0.053 (0.005)	0.107 (0.025)	0.066 (0.005)	0.053 (0.005)	0.048 (0.006)	0.194 (0.011)

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Table F.2 – Continued from previous page

	Magdalou and Nock			Cowell			Almås et al.
	r = 0 (Baseline)	r = 1	r = 2	r = 0	r = 1	r = 2	
2010	0.071 (0.005)	0.048 (0.006)	0.108 (0.035)	0.058 (0.006)	0.048 (0.006)	0.044 (0.006)	0.186 (0.013)
2012	0.130 (0.010)	0.069 (0.009)	0.185 (0.062)	0.094 (0.009)	0.069 (0.009)	0.064 (0.011)	0.224 (0.017)

**Data:** PSID public use dataset and PSID-CNEF.

**Note:** Unfair inequality is calculated as  $D(Y^e || Y^r)$ . This term measures the aggregate divergence between the empirical distribution  $Y^e$  and the norm distribution  $Y^r$ . The norm distribution is given by  $Y_{EOp+FP}^r$ . See also Table 1 for information on the calculation of norm distributions. Divergences are aggregated according to the divergence measure indicated in the column header.