Abstract - We investigate the use of bilateral advance pricing agreements (BAPAs) to resolve transfer pricing disputes between a taxpayer and two tax authorities. BAPAs are designed to protect firms from double taxation while reducing expected compliance costs. We identify settings in which we expect BAPAs to arise and investigate the effect of the program on compliance costs. We show that agreements are more likely to arise when the amount of income potentially subject to double taxation is low and the difference in tax rates between the two countries is high. We also show that the BAPA program can increase compliance costs.

INTRODUCTION

Income earned by a multinational enterprise can be subject to double taxation if multiple tax authorities assert the right to tax the income. In particular, double taxation arises to the extent that governments use different transfer prices to allocate income between countries. In the 1997, 1999, and 2001 Ernst & Young LLP Transfer Pricing Global Surveys, over 80 percent of multinationals identified “double tax relief” as an important international tax issue because transfer pricing adjustments typically result in double taxation (Ackerman and Hobster, 2001).

Governments also have severe difficulties with transfer pricing disputes. Transfer pricing cases are factually intensive investigations featuring both legal and economic analysis and involving both domestic and foreign entities. Billions of dollars of tax revenue are at stake; for example, in 2004 the multinational pharmaceutical firm GlaxoSmithKline was assessed a $2.7 billion U.S. tax deficiency notice arising from transfer pricing disputes involving its 1989–1996 tax years (Sullivan, 2004); the case was settled in 2006 for $3.4 billion, covering tax and interest relating to its 1989–2005 tax years (Nutt, 2006). In 2006, Symantec Corp. disclosed that it was contesting a $1 billion deficiency relating to transfer pricing disputes involving tax years from 2000–2004 (Sikora, 2006). The rapidly growing volume of international transactions between related entities has put the traditional system of international tax compliance under considerable strain.
In response to this mutually unsatisfactory situation, the U.S. has developed and many countries have implemented a model of tax compliance: the bilateral advance pricing agreement (BAPA). In a BAPA, the taxpayer approaches the governments prior to engaging in a transaction with a foreign related party to negotiate a mutually acceptable transfer price.\(^1\) The taxpayer provides detailed information regarding the proposed transaction to the governments. In return, the governments commit to a transfer price that ensures that the taxpayer will not be subject to double taxation.\(^2\)

Korb (2004) reports that the Internal Revenue Service (IRS) currently completes about 60–70 agreements per year, most covering five years or more. While the IRS does not disclose the dollar value of the transactions covered by the agreements, U.S. cross-border trade of merchandise between related parties was $770 billion in 2002. This figure includes neither services nor licensing of intangibles. Williams (1996) describes the BAPA program, the procedures involved, and the costs and benefits of the program. Ring (2000) also describes the program and evaluates it in terms of legal theories of administrative law. Ackerman (2001) examines how the program has evolved into a more standardized format over time. Sheppard (2005) argues that the United States tax authorities are the most eager to complete a BAPA, and make substantial concessions to other countries in order to come to an agreement. The IRS issues annual reports describing the program and providing some descriptive statistics regarding the number of agreements and length of time it takes to complete an agreement (U.S. Department of the Treasury, 2006).

In this paper, we investigate the use of BAPAs using a game-theoretic approach to tax compliance. We examine two research questions. First, under what circumstances do we expect firms and governments to enter into a BAPA, and under what circumstances do we expect firms to follow the traditional approach involving auditing and litigation? Second, what are the effects of the BAPA program on aggregate tax compliance costs, which we define to be the sum of expected audit costs and the cost of negotiating and implementing the BAPA?

With respect to our first research question, we first find that a reduction in aggregate tax compliance costs is a necessary but not sufficient condition for a BAPA to occur. The fact that by definition a BAPA requires the parties to agree upon a single transfer price can make an agreement infeasible even though aggregate tax compliance costs would be lower with an agreement. Second, we find that a BAPA is more likely to be used when the amount of income subject to double taxation because of disagreement over the transfer price is low. Although the rationale for the BAPA program is to provide a mechanism by which the firm can avoid double taxation, the ability of the firm and the tax authorities of the countries to find a mutually acceptable transfer price decreases as the amount of income potentially subject to double taxation increases. Third, we find

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\(^1\) In practice, all the facts (e.g., future production costs) regarding the transaction are rarely known. The agreement generally involves the choice of a transfer price method and a set of procedures for implementing that method. For expositional convenience only, we refer to the agreement as an agreement over the transfer price itself.

\(^2\) Taxpayers also have the option of entering into a unilateral advance pricing agreement in which the agreement is between the taxpayer and one of the two tax authorities. Unilateral agreements do not necessarily protect taxpayers from double taxation. However, a unilateral agreement between the taxpayer and the country with the higher tax rate is likely to be similar to the outcome under a BAPA as long as the tax rate of the country with the lower tax rate is sufficiently low, because then the interests of the taxpayer and the country with the lower tax rate are closely aligned.
that BAPAs occur less frequently when the tax rates of the two countries are similar. Expected audit costs are lower when the tax rates are similar because the country with the higher tax rate does not have to audit as frequently to deter the taxpayer from shifting its taxable income toward the lower tax rate country via its choice of transfer price. When expected audit costs are lower, there is less to gain from a BAPA and so it is used less frequently.

With respect to our second research question, we find that the effect of the BAPA program on compliance costs depends on whether the absence of a BAPA reveals information about the taxpayer that influences the governments’ audit decisions. Even if the tax authorities can credibly commit not to use disclosures made by the taxpayer in the BAPA process in other tax compliance proceedings, the absence of a BAPA may reveal taxpayer information. If the absence of a BAPA reveals no taxpayer information, the existence of the program reduces aggregate compliance costs. However, if the absence of a BAPA does reveal taxpayer information, the existence of the program can increase aggregate tax compliance costs by increasing the frequency with which they audit the taxpayer, compared to what they would have done if the BAPA program had not existed. Despite the increase in expected total compliance costs in these cases, the BAPA program increases the expected payoff (taxes collected minus compliance costs) of the country with the higher tax rate. Therefore, we expect that high tax countries will most aggressively promote the use of BAPAs. This is consistent with the United States being the most eager to negotiate a BAPA, as Sheppard (2005) suggests, if the United States has the higher tax rate. Sheppard also suggests that the U.S. eagerness to negotiate a BAPA implies that they give away too much in the negotiations. However, our model highlights the importance of audit costs when evaluating a BAPA, a factor that Sheppard does not consider. Since a BAPA can yield a significant reduction in compliance costs, giving away a lot in the transfer price negotiations does not imply incompetence; it may be necessary in order to get the other parties (the low tax country and the firm) to accept a BAPA.

In the next section we review the literature that relates to our paper. The third section presents the model. The fourth section characterizes the equilibrium behavior of the taxpayer and the two tax authorities when a BAPA does not occur. The fifth section characterizes the equilibrium behavior of the taxpayer and the two tax authorities regarding their decision to enter a BAPA, and the corresponding equilibrium behavior in absence of a BAPA. In the sixth section we discuss the implications of our results. We also examine the effects of the BAPA program on tax compliance costs. The seventh section concludes.

RELATED LITERATURE

Tomohara (2004) is the only other paper to investigate formally the economic effects of BAPAs. That paper considers a complete information setting in which the firm and both governments agree upon a transfer price, after which pretax profits and taxes are derived endogenously. The focus of that paper is on the firm’s production decisions in the presence of a BAPA, and the productive efficiency losses that arise as the firm alters its production decisions in response to the transfer price and the tax rate differences between the two countries. In contrast, our paper considers an incomplete information setting in which countries must incur costs to overcome their informational disadvantage. We consider a given set of transactions, so that the firm’s pretax income from these transactions is fixed, and focus on whether the taxpayer and governments will enter into a BAPA, how the presence
of the BAPA program affects their reporting and auditing decisions, and how the BAPA program affects total tax compliance costs.

Our paper relates to several topics that have been thoroughly explored in the tax literature. Many papers have modeled the interaction between the taxpayer and the tax authority as a game between a wealth–maximizing taxpayer and a tax authority trying to maximize government revenues net of audit costs (Graetz, Reinganum, and Wilde, 1986; Reinganum and Wilde, 1986; Beck and Jung, 1989; Sansing, 1993; Rhoades, 1997; Rhoades, 1999; Mills and Sansing, 2000; Feltham and Paquette, 2002). De Waegenaere, Sansing, and Wielhouwer (2006) extend the standard tax compliance model to a multinational setting in which the taxpayer interacts with two tax authorities enforcing the transfer pricing rules of countries with different tax rates. We use a similar approach to determine the equilibrium behaviors of the taxpayer and the tax authorities in case the parties do not enter into a BAPA.

Our study of the BAPA process relates to research on alternatives to litigation as a means of resolving legal disputes. Samuelson (1991) investigates the use of final–offer arbitration under asymmetric information. Sansing (1997) applies a variation of Samuelson’s model to a tax litigation setting. He finds settings in which the taxpayer and the tax authority will only agree to arbitration for certain realizations of their private information. Our results are similar in that whether the parties enter into a BAPA depends on the taxpayer’s private information.

As BAPAs are used to resolve tax transfer pricing disputes, our paper relates to the transfer pricing literature as well. In particular, the fact that the two countries prefer different tax transfer prices that would result in double taxation in absence of a BAPA gives the taxpayer the incentive to enter into a BAPA. It also characterizes the conditions under which the BAPA process will break down. Other papers that have investigated the distributional and efficiency effects when countries use asymmetric tax transfer pricing methods include Halperin and Srinidhi (1987) and Elitzur and Mintz (1996). There is a much larger literature on distributional and efficiency effects of tax transfer prices when both countries use the same transfer price method; Baldenius, Melumad, and Reichelstein (2004) review this literature in their paper.

THE MODEL

A firm $F$ earns income $W > 0$ from one of its business activities that takes place in two different countries, $H$ and $L$. The two countries impose tax rates of $t_H$ and $t_L$, respectively, $t_H \geq t_L$. The business uses an asset created or developed in one of the countries. The division of the income from the business between the two countries depends in part on the transfer price associated with the use of the asset. For example, a subsidiary operating in country $L$ and a parent operating in country $H$ could generate income of $W$ in a business that uses a patent that was developed by the parent corporation. The division of the income $W$ between the parent and subsidiary depends in part on the royalty that the subsidiary pays to the parent for the use of the parent’s patent.

We assume that both countries adhere to the arm’s–length standard as their transfer pricing principle, but apply this principle in different ways. Different interpretations of the same principle can arise if different methods (e.g., comparable uncontrolled price, cost–plus, comparable profit) of determining an arm’s–length transfer price yield different prices, which often occur with complicated or non–routine transactions. In addition, the countries may use different ways of implementing a given method. If both countries know all the facts that could affect the transfer price, then under $H$’s transfer price, the
income taxed by $H$ is $y_H W$, $0 \leq y_H \leq 1$, and the income taxed by $L$ is $(1 - y_H) W$; under $L$’s transfer price, the income taxed by $L$ is $y_L W$, $0 \leq y_L \leq 1$, and the income taxed by $H$ is $(1 - y_L) W$. We assume that the countries’ preferred interpretations of the transfer pricing principle give rise to double taxation, i.e., $y_H + y_L \geq 1$. We let $x$ represent the fraction of income that would be subject to double taxation if each country were to apply its own transfer price, so $x = y_H + y_L - 1 \geq 0$. We assume that $(y_H, y_L)$ and $W$ are realizations from a probability distribution, that $x$ has strictly positive probability density on the interval $[0, 1]$ and that $x$ and $W$ are independent. We let $E[x] = \mu$. All aspects of the model are common knowledge except for the realizations of $(y_H, y_L)$ and $W$. The firm knows whether the transfer price involves an undifferentiated “commodity” product with a readily ascertainable market price, in which case both countries would use similar prices for tax purposes (i.e., $y_H = 1 - y_L$) and, thus, the fraction of income subject to double taxation $x = y_H + y_L - 1$ would be close to zero. The firm also knows whether the transfer price involves a difficult-to-value intangible asset (e.g., a drug patent), in which case $y_H$ and $1 - y_L$ could be very different, so that $x$ could be close to one. This implies that there would typically be substantial information asymmetry between the firm and the tax authorities regarding the value of $(y_H, y_L)$. For simplicity, we assume that the firm knows the realization of $(y_H, y_L)$ and, thus, the fraction of income that will be subject to double taxation, $x$, whereas the governments only know the probability distribution of $(y_H, y_L)$. The firm privately observes the income $W$ generated by the business when it is realized.

If the firm follows the traditional approach of filing its tax returns, potentially followed by audit and litigation, it chooses a transfer price that yields an internally consistent income division $(r W, (1 - r) W)$, where $r$ is the fraction of income from the business reported to $H$ and $1 - r$ is the fraction of income reported to $L$. We restrict attention to internally consistent reports because we view the problem that BAPAs are designed to solve as being one of tax avoidance through income shifting, rather than tax evasion through underreporting. The GlaxoSmithKline transfer pricing case illustrates the importance of disputes over tax avoidance where tax evasion is not an issue. Sullivan (2004) reviews the U.S. Tax Court filings, and finds that GlaxoSmithKline reported $3.9$ billion of its $12.7$ billion worldwide profit between 1989 and 1996 to the U.S. and $8.8$ billion to the U.K. The IRS deficiency notice proposed an allocation of $\$10.3$ billion to the U.S. and $\$2.4$ billion to the U.K. The total worldwide income of $\$12.7$ billion is not in dispute. Using different transfer prices for the same transaction in an attempt to report less than $W$ of taxable income from the business is a form of tax evasion that can trigger severe civil and criminal penalties. Our approach assumes that the (unmodeled) penalties associated with detected tax evasion are sufficiently severe to deter it.

We let $r = \alpha y_H + (1 - \alpha)(1 - y_L)$, where $\alpha$ is the weight that $F$ puts on country $H$’s transfer price and $1 - \alpha$ is the weight placed on $L$’s transfer price. Any $0 \leq \alpha \leq 1$ is a reasonable interpretation of the arm’s-length principle. In equilibrium, $\alpha$ is never less than zero nor greater than one. We illustrate the firm’s reporting problem in Figure 1.

Each country has the option to audit the related party transaction. The cost of an audit is $K > 0$ per dollar of income associated with the business. The country that audits receives the difference between the reported income and the income measured under that country’s tax transfer price, which is equal to $(1 - \alpha) x W t_H$ for country $H$ and $\alpha x W t_L$ for country $L$. Country $H$ ($L$) conducts an audit of the taxpayer’s related party transaction with
probability $\delta_H(\delta_L)$. We assume that $K < \mu_{t'H'}$, which ensures that auditing is not so costly as to deter $H$ from auditing if $F$ were to report all of the income from the business in accordance with $L$’s transfer price.

We assume that the tax authorities cannot observe the taxpayer’s division of the income from the business without conducting a costly audit, so that the audit probabilities $\delta_H$ and $\delta_L$ cannot depend on $(rW, (1-r)W)$. This assumption diverges from the usual approach in models of individual tax evasion (Reinganum and Wilde, 1986; Sansing, 1993; Erard and Feinstein, 1994; Rhoades, 1999) in which the audit decision depends on the taxpayer’s report. The different assumptions reflect the differences in information environments in which individual tax evasion and corporate tax avoidance take place. Individual tax evasion occurs in a setting in which a large population of taxpayers file relatively simple tax returns in an environment with a high level of third party reporting of many items of income (such as wages, dividends, and interest.) Given the size of the population, the extent of third party reporting, and the simplicity of the tax return, models of individual tax evasion typically assume that the tax authority is able to costlessly use the report to infer the likelihood of tax evasion before deciding whether to conduct a costly audit. In contrast, our setting features a much smaller population of corporate taxpayers filing very complicated returns with little third party reporting. For the setting we consider, the income $rW$ reported to $H$, for example, is the sum of income from a large number of transactions from the business associated with the related party transaction. In addition, these transactions are typically aggregated with a large number of other transactions in an extensive report. For example, General Electric’s 2006 U.S. corporate income tax return was 24,000 pages long. We, therefore, find it implausible that the tax authorities would be able to observe or infer information regarding

Note: The bold line is the range of reports $(r,1-r)$ for which $\alpha \in [0,1]$.

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3 Large taxpayers are audited every year. We emphasize that $\delta_H$ and $\delta_L$ are the probabilities that a particular related party transaction is audited, not the probability that the taxpayer is audited. The predictions of the strategic auditing model were supported in a study of large taxpayers (Mills and Sansing, 2000).
the value of $r$ and $W$ without conducting a costly audit. Therefore, neither $\delta_H$ nor $\delta_L$ can depend on the taxpayer’s division of income from the business activity associated with the related party transaction.

As an alternative to dealing with the conflict in the countries’ transfer prices via the audit and litigation process, the firm can deal with the conflict in the countries’ transfer prices cooperatively via a BAPA. In a BAPA, the firm approaches countries $H$ and $L$ regarding the possibility of agreeing to a transfer price resulting in an allocation of the income in which, if income $W$ is ultimately realized, $zW$ is allocated to $H$ and $(1 – z)W$ is allocated to $L$. This allocation ensures that double taxation does not occur. The cost to all parties of negotiating and implementing a BAPA is $C \geq 0$ per dollar of expected income from the business.

An important practical issue is whether the governments have the option of acting in bad faith by engaging in BAPA negotiations solely to obtain information about the values of $W$, $y_H$, and $y_L$, then using this information against the taxpayer in an adversarial audit process. Taxpayers, of course, would not be willing to negotiate under such circumstances. To guard against this outcome, Sections 9.03 and 9.04 of Rev. Proc. 91–22 [1991 C.B. 526] bars the Internal Revenue Service from using taxpayer disclosures in any proceeding not covered by an advance pricing agreement. In addition, the tax authority’s long–term interest in preserving the BAPA program gives it an incentive not to behave opportunistically by negotiating in bad faith. Accordingly, we model this tension between the tax authority’s short–term interest in maximizing its payoff from this taxpayer and its long–term interest in the BAPA program by assuming that the tax authority will acquiesce to a proposed BAPA that increases its expected payoff compared to what its expected payoff would have been had the taxpayer not sought to negotiate a BAPA.

The interaction among the players proceeds as follows. First, the firm observes $(y_H, y_L)$, and rationally anticipates whether a BAPA is feasible, i.e., whether there exists a transfer price and an allocation of the BAPA cost such that the resulting expected payoffs for the firm and for each of the governments are weakly higher than without the BAPA. If a BAPA is feasible, the negotiation process reveals the realizations of $(y_H, y_L)$, and the parties enter into a BAPA. $F$ subsequently observes $W$, then files its tax returns in a manner consistent with the transfer price agreed in the BAPA, and $H$ and $L$ accept the returns as filed. If a BAPA is not feasible, the firm observes $W$, files its tax returns without a BAPA, and the countries decide whether to audit the returns. The timeline is illustrated in Figure 2.

Whether a BAPA is feasible thus depends on the players’ behavior in absence of a BAPA. In the next section, we characterize the players’ Nash equilibrium strategies in that reporting game. Because the players’ equilibrium strategies in the absence of a BAPA do not depend on the realization of $W$ or on its probability distribution, without loss of generality we assume that both the realization and the expectation of $W$ are equal to one.

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4 The IRS has developed a reputation for acting with restraint in similar settings. For example, it will only seek tax accrual workpapers when investigating listed transactions, even though it has the legal authority to seek the workpapers in many more cases (Tandon, 2006).

5 Given that firms and the governments behave in this manner in general, deviating in any particular instance (e.g., by having the firm not propose a BAPA that is feasible) would decrease the deviating player’s payoff. However, other sets of behaviors also have this property. For example, if firms could collectively boycott the BAPA process for certain parameter values, doing so could alter the governments’ optimal audit strategies in the reporting game.
Figure 2. Game Timeline

$t_0$: Firm observes $(\gamma, y)$.  
$t_1$: Firm determines whether a feasible BAPA exists.  
$t_2$: $(\gamma, y)$ is revealed; parties agree on a transfer price that divides income of $W$ $(\varepsilon W, (1-\varepsilon)W)$.  
$t_3$: $W$ is realized; BAPA payoffs are realized.  
$t_4$: Governments decide whether to audit.  
$t_5$: Audit and litigation payoffs realized.

Yes

No

The reporting game
THE REPORTING GAME

In this section, we characterize the behavior of the players in case they do not enter into a BAPA. The expected payoff to country $H$ given a particular realization $(y_{Ht}, y_{Lt})$ and reported income $r$ is

$$[1] \quad P_H = t_H r + \delta_H [t_H (y_{Ht} - r) - K].$$

The expected payoff to country $L$ given a particular realization $(y_{Ht}, y_{Lt})$ and reported income $1 - r$ is

$$[2] \quad P_L = t_L (1 - r) + \delta_L [t_L (y_{Lt} - (1 - r)) - K].$$

The expected payoff to the firm given a particular realization $(y_{Ht}, y_{Lt})$ and reported income division $(r, 1 - r)$ is

$$[3] \quad P_f = -t_H [r + \delta_H (y_{Ht} - r)] - t_L [(1 - r) + \delta_L (y_{Lt} - (1 - r))].$$

We define a Nash equilibrium for the reporting game as a reporting strategy $r(y_{Ht}, y_{Lt}) = \alpha y_{Ht} + (1 - \alpha)(1 - y_{Lt})$ that maximizes the firm’s expected payoff given the equilibrium audit strategies of $H$ and $L$; an audit probability $\delta_H$ for country $H$ that maximizes its expected payoff given the firm’s reporting strategy; and an audit probability $\delta_L$ for country $L$ that maximizes its expected payoff given the firm’s reporting strategy.

Proposition 1 establishes the equilibrium strategies of the players in the reporting game. The equilibrium strategies depend on the expected value of $x = y_{Ht} + y_{Lt} - 1$, the fraction of income subject to double taxation. Because the reporting game only occurs if the players have not entered into a BAPA, the countries may be able to infer information regarding the value of $x$ from the fact that the firm is filing a tax return without a BAPA. We let $\hat{\mu}$ denote the conditional expectation of $x$, given that a BAPA is not entered into, i.e., $\hat{\mu} = E[x | \text{No BAPA}]$. We assume in the following proposition that $\hat{\mu} \geq \mu$; we will show in the proofs of Propositions 2, 3 and 4 that this assumption indeed holds in this game because in equilibrium $\hat{\mu}$ is always weakly greater than $\mu$.

There are two equilibria, depending on $K$, tax rates and $\hat{\mu}$. If the cost of auditing ($K$) is sufficiently low compared to the tax rates and $\hat{\mu}$, the expected fraction of income that is subject to double taxation given that the parties did not enter a BAPA, both $H$ and $L$ audit all reports and $F$ reports part of its income according to $L$’s transfer price and part according to $H$’s transfer price. We refer to this as the heavy audit equilibrium. If the cost of auditing is sufficiently high, country $L$ never audits and country $H$ uses a mixed audit strategy that makes $F$ indifferent regarding its reports. The firm reports part of its income according to $L$’s transfer price and part according to $H$’s transfer price so as to make $H$ indifferent between auditing and not auditing the report. We refer to this as the light audit equilibrium.

**Proposition 1**

Assume that $\hat{\mu} \geq \mu$.

(a) If $K \leq \mu t_H t_L / (t_H + t_L)$, then the following strategies constitute a Nash equilibrium:

(i) $F$ reports $r = \alpha y_{Ht} + (1 - \alpha)(1 - y_{Lt})$ to country $H$ and $1 - r = (1 - \alpha)y_{Lt} + \alpha(1 - y_{Ht})$ to country $L$, where $\alpha \in [K / (\hat{\mu}_H - K)]$, $\hat{\mu}_H / (\hat{\mu}_H - K)$; 
(ii) $H$ always audits, i.e., $\delta_H = 1$; 
(iii) $L$ always audits, i.e., $\delta_L = 1$.

(b) If $K \geq \mu t_H t_L / (t_H + t_L)$, then the following strategies constitute a Nash equilibrium:

(i) $F$ reports $r = \alpha y_{Ht} + (1 - \alpha)(1 - y_{Lt})$ to country $H$ and $1 - r = (1 - \alpha)y_{Lt} + \alpha(1 - y_{Ht})$ to country $L$, where $\alpha = (\hat{\mu}_H - K) / (\hat{\mu}_H)$; 
(ii) $H$ audits with probability $\delta_H = (t_H - t_L) / t_H$; 
(iii) $L$ never audits, i.e., $\delta_L = 0$.

In the heavy audit equilibrium, the cost of auditing is so low relative to the countries’ tax rates and the expected fraction...
of income subject to double taxation that one of the countries will have an optimal strategy of always auditing. To deter the firm from reporting all of its income according to the rules of that country, the other country should also audit all reports. Given that both countries will audit the report, the only equilibrium reporting strategy for the firm is to report its income in a way that provides both countries with an incentive to audit.

In the light audit equilibrium, the cost of auditing is sufficiently high that country $L$ has an optimal strategy of never auditing. Country $H$ audits just enough to deter the firm from reporting all of its income according to country $L$’s transfer price. This audit probability is decreasing in the tax rate ratio $t_L/t_H$. The firm in turn reports just enough of its income in accordance with country $H$’s transfer price to deter it from auditing all reports. The fraction of the income reported in accordance with country $H$’s transfer price is increasing in $t_H$ and $\hat{\mu}$ and decreasing in the audit cost $K$.

BILATERAL ADVANCE PRICING AGREEMENTS

In this section, we determine the conditions under which the players will enter a BAPA, and their optimal response in the absence of a BAPA.

In a BAPA, the firm approaches countries $H$ and $L$ regarding the possibility of agreeing to a transfer price resulting in an allocation of the income in which $z$ is allocated to $H$ and $1 - z$ is allocated to $L$. We allow any nonnegative allocation of the cost $C$ of negotiating and implementing the BAPA among the three players. The players’ payoffs before the allocation of $C$ are

\[ A_F = -zt_H - (1 - z)t_L, \]

\[ A_H = zt_H, \]

\[ A_L = (1 - z)t_L. \]

We have defined a feasible BAPA to be a BAPA in which the allocation of income $(z, 1 - z)$ and a division of the total costs $C$ are such that each player receives a (weakly) higher payoff under the BAPA than without the BAPA. Because the firm only requests a BAPA negotiation when it knows that a BAPA is feasible, the value of $(y_H, y_L)$ is never revealed when the firm does not enter a BAPA. Therefore, in absence of a BAPA, the players’ strategies are as described in Proposition 1, and a BAPA is feasible if and only if the following four conditions are satisfied:

\[ A_F \geq P_F, \quad A_H \geq P_H, \quad A_L \geq P_L, \quad \text{and} \]
\[ A_F + A_H + A_L - C \geq P_F + P_H + P_L, \]

with $P_H$, $P_L$ and $P_F$ given by [1], [2], and [3] for the equilibrium strategies shown in Proposition 1.

We refer to a BAPA as efficient if condition [8] is satisfied. This condition states that a BAPA is efficient if social welfare (the sum of all payoffs) increases. Because tax payments are zero–sum wealth transfers between the three players, this condition is satisfied if the BAPA cost $C$ is less than the aggregate expected audit cost when the players do not enter a BAPA.

The feasibility of a BAPA depends on how the game would be played in absence of a BAPA, i.e., whether the players would play the heavy audit equilibrium or the light audit equilibrium. The efficiency condition [8] is satisfied if the BAPA cost $C$ is less than the audit cost of $2K$ under the heavy audit equilibrium and less than the expected audit cost of $K(t_H - t_L)/t_H$ under the light audit equilibrium.\(^6\) The

\[^6\] We make no assumptions regarding the relative sizes of the BAPA cost $C$ and the audit cost $K$. On the one hand, the tax authorities can and do impose costly information demands on the firm. On the other hand, the BAPA process is a cooperative process, which suggests that acquiring information is relatively less costly. Moreover, in a bilateral process, each country automatically receives the information revealed to the other country.
individual feasibility conditions in [7] will sometimes depend on $x$, the fraction of income potentially subject to double taxation.

Whether a BAPA is feasible thus depends on the players’ behavior in the absence of a BAPA and, sometimes, on the value of $x$. Because the players enter a BAPA whenever one is feasible, the tax authorities may be able to infer information regarding the value of $x$ from the fact that the firm is filing a tax return without a BAPA. Therefore, the players’ equilibrium behavior in absence of a BAPA also depends on whether and when they enter a BAPA. In Propositions 2, 3 and 4, we characterize the players’ equilibrium decision to enter a BAPA, and the corresponding equilibrium behavior in absence of a BAPA.

Proposition 2 characterizes behavior when $C$ is sufficiently high. When $C \geq 2K$, the cost of a BAPA is so high that a BAPA is not efficient, even if the players would choose their heavy audit strategies in the reporting game. Because efficiency is a necessary condition for feasibility, a feasible BAPA does not exist. Therefore, the players never enter a BAPA, so that the tax authorities cannot infer any information about $x$ from the absence of a BAPA. The equilibrium then depends on $K$, as described in Proposition 1 with $\mu = \mu$, because $\mu = E[x | \text{No BAPA}]$.

**Proposition 2**

If $C \geq 2K$, then the Nash equilibria are:

(a) if $K \leq \mu t_L t_1/(t_H + t_I)$ then the players never enter a BAPA, and choose their heavy audit strategies in the reporting game;

(b) if $K \geq \mu t_L t_1/(t_H + t_I)$ then the players never enter a BAPA, and choose their light audit strategies in the reporting game.

Proposition 3 characterizes the equilibria in which $C$ takes on intermediate values ($K(t_H - t_I)/t_H \leq C \leq 2K$). For these values of $C$, a BAPA is not efficient when the players would choose their light audit strategies, because the cost of the BAPA then exceeds the aggregate expected audit costs. If the players never enter a BAPA, then no information is revealed from the absence of a BAPA, so that $\mu = \mu$. Therefore, it follows from Proposition 1(b) that light auditing is the optimal response for all $K \geq \mu t_L t_1/(t_H + t_I)$.

If the players would choose their heavy audit strategies in the reporting game, a BAPA is efficient because the cost of a BAPA is less than the aggregate audit costs ($C < 2K$). However, the feasibility of a BAPA depends on $x$, the fraction of income that is potentially subject to double taxation ($x = y_H + y_L - 1$). If $x$ is too high, a BAPA is not feasible. When $x$ is high, the gains to $H$ and $L$ from double taxation under the heavy audit equilibrium are high. Therefore, a transfer price leading to an income allocation $(z, 1 - z)$ that eliminates double taxation and makes both $H$ and $L$ better off under a BAPA than they would be under the heavy audit equilibrium does not exist, even if $F$ were to bear all of the cost of the BAPA. The largest value of $x$ for which a BAPA is feasible, which we denote $x^*$, is $x^* = K(t_H + t_I)/(t_H t_I)$.

It is in this situation that the governments update their beliefs regarding $x$ given the absence of a BAPA. If the players enter a BAPA for all $x \leq x^*$, then heavy auditing is the optimal response in the absence of a BAPA for all $K \leq t_H t_L/(t_H + t_I)$.

**Proposition 3**

If $(K(t_H - t_I)/t_H \leq C \leq 2K)$, then the Nash equilibria are:

(a) $K \leq t_H t_L/(t_H + t_I)$, then the players enter a BAPA if $x \leq x^*$, and choose their heavy audit strategies in the reporting game if $x > x^*$;

(b) if $K \geq \mu t_L t_1/(t_H + t_I)$, then the players never enter a BAPA and choose their light audit strategies in the reporting game.

An important implication of Proposition 3 is that either type of equilibrium
(i.e., BAPA for $x \leq x^*$ and heavy audit otherwise, or never BAPA and light audit) can arise when $\mu t_H (t_H + t_L) \leq K \leq t_H t_L / (t_H + t_L)$ and $K (t_H - t_L) / t_H \leq C \leq 2K$. If the firm anticipates that the countries will choose their heavy audit strategies, then it should seek a BAPA when $x \leq x^*$, in which case heavy audit strategies are appropriate in the absence of a BAPA. On the other hand, if the firm anticipates that the countries will choose their light audit strategies, it should never seek a BAPA, in which case light audit strategies are appropriate. While we offer no prediction regarding which equilibrium will be played, aggregate payoffs are higher in the “no BAPA/light audit” equilibrium, while country $H$’s payoff is greater in the “some BAPA/heavy audit” equilibrium. This occurs because country $H$ has an expected payoff equal to $t_H E[y_H] - K$ in either the heavy or light audit equilibrium and has a minimum expected payoff of $t_H E[y_H] - K$ in a BAPA. Therefore, we predict that high–tax countries will aggressively encourage the use of BAPAs to resolve tax transfer pricing disputes.

Proposition 4 characterizes the Nash equilibria when $C$ is low ($C \leq K (t_H - t_L) / t_H$). A BAPA is always efficient in that case, because the cost of the BAPA is lower than the aggregate expected audit costs, even under the light audit equilibrium. When $K \leq t_H t_L / (t_H + t_L)$, a BAPA is only feasible if $x \leq x^*$, in which case the players would choose their heavy audit strategies in the absence of a BAPA. If $K \geq t_H t_L / (t_H + t_L)$, the players would choose their light audit strategies in the absence of a BAPA, and a BAPA is feasible for all $x$. Therefore, the players will negotiate a BAPA for all realizations of $x$, and the reporting game is never played in this case. This equilibrium is supported by the off–equilibrium path beliefs of the tax authorities of $H$ and $L$ that $\hat{\mu} = 1$.

Proposition 4
If $C \leq K (t_H - t_L) / t_H$, then the Nash equilibria are:

(a) if $K \leq t_H t_L / (t_H + t_L)$, then the players enter a BAPA if $x \leq x^*$, and choose their heavy audit strategies in the reporting game if $x > x^*$;
(b) if $K \geq t_H t_L / (t_H + t_L)$, then the players always enter a BAPA, and the off–equilibrium path beliefs are that $\hat{\mu} = 1$.

Figure 3 illustrates the equilibrium outcomes for $t_H = 40\%$, $t_L = 20\%$, and $\mu = 0.5$.

DISCUSSION AND ANALYSIS

Implications

We first consider three implications of our analysis. First, a reduction in expected tax compliance costs is a necessary but not sufficient condition for a BAPA to occur. This occurs because the efficiency of a BAPA depends on the difference between the cost of a BAPA and the aggregate expected audit costs of the countries, whereas the feasibility of a BAPA sometimes also requires that the fraction of income potentially subject to double taxation be low. All feasible BAPAs are efficient, but the converse does not hold.

Second, the players are more likely to enter into a BAPA when the amount of income subject to double taxation because of asymmetric transfer prices is low, i.e., $x \leq x^*$. Although the rationale for the BAPA program is to provide a mechanism by which the firm can avoid double taxation, the fact that by definition a BAPA requires the parties to agree upon a single transfer price makes the procedure infeasible when the countries’ tax laws imply large differences in their respective transfer prices.

Third, when the cost of auditing is sufficiently high (i.e., $K \geq t_H t_L / (t_H + t_L)$), BAPAs occur less frequently when the tax rates of the two countries are similar. When $K$ is high and the tax rates are similar, the firm has little incentive to shift income to the lower–tax country, the low–tax country
does not audit (because of the high audit cost relative to the expected increase in revenue from double taxation), and the high–tax country audits infrequently (because little auditing is required to deter the firm from reporting using the low–tax country’s transfer price.) As a consequence, the expected audit costs are low, so that a BAPA is only efficient when the cost $C$ of negotiating and implementing the BAPA is very low. The BAPA is, therefore, not well–suited to resolve double taxation problems when the two countries have similar tax rates and the cost $K$ of auditing is high. BAPAs are more likely to be efficient when the firm has a stronger incentive to shift income to the lower–tax country due to large differences in tax rates. This high difference in tax rates increases the incentive to shift income, thereby increasing expected audit costs, making the BAPA relatively more attractive.

**Efficiency Implications of the BAPA Program**

We complete our analysis by comparing the cost of tax compliance (tax auditing costs or BAPA negotiation and implementation costs) when the BAPA program exists to what the cost would have been in absence of the program. We consider two cases. First, when either $K$ is very low ($K \leq \mu t_H t_L / (t_H + t_L)$), very high ($K \geq t_H t_L / (t_H + t_L)$) or BAPA costs are very high ($C \geq 2K$), the reporting game is unaffected by the BAPA program. When BAPAs are entered into, all the players’ payoffs weakly improve, because all feasible BAPAs are efficient.

However, for intermediate values of $K$, ($\mu t_H t_L / (t_H + t_L) \leq K \leq t_H t_L / (t_H + t_L)$) and
C ≤ 2K, the very existence of the BAPA program can change the audit strategies that occur in the reporting game. When \( K(t_H - t_L)/t_H ≤ C ≤ 2K \), Proposition 3(a) shows that an equilibrium exists in which the players choose their heavy audit strategies when a BAPA does not occur, but would have chosen their light audit strategies if the BAPA program did not exist. This occurs because the tax audit authorities of countries \( L \) and \( H \) infer from the absence of a BAPA that the potential level of double taxation is high. They infer important taxpayer information from the absence of a BAPA even though they learn nothing directly about the taxpayer from the BAPA process itself.

As Proposition 3(b) shows, this inefficiency need not arise, because it is still an equilibrium for the firm to not seek a BAPA and for the players to choose their light audit strategies in the reporting game. Because the "no BAPA and light audit" equilibrium features lower aggregate compliance costs than the "some BAPA and heavy audit" equilibrium, at least one player will prefer the equilibrium without BAPAs. However, country \( H \)'s payoffs are higher in the equilibrium in which a BAPA arises for sufficiently low \( x \). We expect high tax rate countries to be the most aggressive in promoting BAPAs for this reason; whether the BAPA program creates an inefficiency depends on whether the high–tax country succeeds in getting the players to play its preferred equilibrium.

When BAPA costs are very low (\( C ≤ K(t_H - t_L)/t_H \) and \( \mu t_H t_L/(t_H + t_L) ≤ K ≤ t_H t_L/(t_H + t_L) \)), the efficiency implications of the BAPA

![Figure 4. Effects of BAPA program on tax compliance costs for \( t_H = 40\% \), \( t_L = 20\% \), \( \mu = 0.5 \).](image)

Note: D indicates that compliance costs are decreasing; I indicates that compliance costs are increasing; and U indicates that compliance costs are unaffected.
program are mixed. When the potential for double taxation is low ($x \leq x^*$), the players enter into a BAPA, which is the efficient outcome. When the potential for double taxation is high, however, a BAPA is not feasible and players choose their heavy audit strategies, but would have chosen their light audit strategies if the BAPA program did not exist. Therefore, total tax compliance costs increase.

Proposition 5 summarizes the implications of the existence of a BAPA program on total tax compliance costs.

**Proposition 5**
The existence of a BAPA program
(a) decreases compliance costs for $x \leq x^*$ if $K \leq \mu t_H t_L / (t_H + t_L)$ and $C \leq 2K$;
(b) decreases compliance costs if $K \geq t_H t_L / (t_H + t_L)$ and $C \leq K(t_H - t_L) / t_H$;
(c) increases compliance costs if $K \geq t_H t_L / (t_H + t_L)$ and $C \leq 2K$ when the players choose the equilibrium strategies described in Proposition 3(a), and has no effect on compliance costs when the players choose the equilibrium strategies described in Proposition 3(b);
(d) increases compliance costs for $x > x^*$ and decreases compliance costs for $x \leq x^*$ if $\mu t_H t_L / (t_H + t_L) \leq K \leq t_H t_L / (t_H + t_L)$ and $C \leq K(t_H - t_L) / t_H$.

For combinations of parameter values not mentioned in Proposition 5, the BAPA program has no effect on tax compliance costs. We illustrate the possibilities described in Proposition 5 in Figure 4.

**CONCLUSIONS**

We have modeled the resolution of tax transfer pricing disputes as a strategic game between the firm and two governments. BAPAs have emerged as an alternative means of resolving these disputes. BAPAs are designed to protect firms from double taxation and reduce tax compliance costs.

Our analysis yields three insights into when such agreements will arise. First, a reduction in expected tax compliance costs is a necessary but not sufficient condition for a BAPA to occur. Second, BAPAs are less likely to occur when the amount of income potentially subject to double taxation because of asymmetric transfer prices is high. Third, BAPAs occur less frequently when the tax rates of the two countries are similar.

We also investigate the effect of the BAPA program on tax compliance costs. We find that the program can increase tax compliance costs. This occurs when the absence of a BAPA conveys information to the tax authorities, inducing them to increase the level of auditing over what it would have been in the absence of a BAPA program. Although this is a socially inefficient outcome, it increases the expected payoff of the high–tax country. Therefore, we expect that countries with the highest tax rates will most aggressively promote the use of BAPAs.

**Acknowledgments**

We thank Rajesh Aggarwal, David Harris, Reed Smith, and two anonymous reviewers for helpful comments.

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APPENDIX

Proof of Proposition 1

(a) \( K \leq \mu t_H/y_H - (t_H + t_L) \)

(i) Given the audit strategies \( \delta_H = 1 \) and \( \delta_L = 1 \), the firm is indifferent over all possible values of \( \alpha \). Substituting \( r = \alpha y_H + (1 - \alpha)(1 - y_L) \) into equation (3) yields

\[
P_f = -t_H(1 - y_L) - t_L y_H - t_H x,
\]

which does not depend on \( \alpha \).

(ii) Given the firm’s equilibrium reporting strategy \( \alpha \in (K/(\hat{\mu}_H), (\hat{\mu}_H - K)/(\hat{\mu}_L)) \), it is optimal for \( H \) to always audit because the expected incremental payoff from auditing is

\[
E[t_H(y_H - r) - K] = E[t_H(1 - \alpha)(y_H - (1 - y_L))]
\]  
\[
- K = E[t_H(1 - \alpha)x] - K,
\]

which is positive because \( \alpha \geq K/(\hat{\mu}_H) \). The assumption that \( K \leq \mu t_H/y_H - (t_H + t_L) \) ensures that an \( \alpha \) exists in the interval \( \alpha \in [K/(\hat{\mu}_H), (\hat{\mu}_H - K)/(\hat{\mu}_L)] \subseteq [0, 1] \).

(b) \( K \geq \mu t_H/t_H + t_L \)

(i) \( H \) is indifferent between auditing and not auditing the report. \( H \)'s incremental payoff from auditing is

\[
E[t_H(y_H - r) - K] = E[t_H(1 - \alpha)(y_H - (1 - y_L))]
\]  
\[
- K = E[t_H(1 - \alpha)x] - K = 0,
\]

because \( \alpha = (\hat{\mu}_H - K)/(\hat{\mu}_L) \). The assumptions that \( K \leq \mu t_H \) and \( \hat{\mu} \geq \mu \) ensure that \( 0 \leq \alpha \leq 1 \).

(ii) \( L \) never audits because its incremental payoff from auditing is:

\[
E[t_L(y_H - (1 - r)) - K] = E[t_L \alpha(y_L - (1 - y_H))]
\]  
\[
- K = E[t_L \alpha x] - K,
\]

which is negative because \( \alpha = (\hat{\mu}_H - K)/(\hat{\mu}_L) \) and \( K \geq \mu t_H/t_H + t_L \).

(iii) Given \( \delta_H = (t_H - t_L)/t_H \) and \( \delta_L = 0 \), \( F \) is indifferent among its reports, because substituting \( r = \alpha y_H + (1 - \alpha)(1 - y_L) \) into equation [3] yields:

\[
P_F = -t_H y_H - t_L(1 - y_H),
\]

which is independent of \( \alpha \). QED

Proof of Proposition 2

(a),(b) Because \( C \geq 2K \), a BAPA is never feasible, which implies \( \hat{\mu} = \mu \). Proposition 1 then implies that the players choose their heavy audit strategies in the reporting game when \( K \leq \mu t_H/t_H + t_L \) and their light audit strategies when \( K \geq \mu t_H/t_H + t_L \).

Proof of Proposition 3

(a) We show that if the players choose their heavy audit strategies in the reporting game, a BAPA is feasible if and only if \( x \leq x^* \), and if the players enter a BAPA if and only if \( x \leq x^* \), the heavy audit strategies are the best response in the reporting game when \( K \leq \mu t_H/t_H + t_L \). Suppose the players choose their heavy audit strategies in the reporting game. Because \( C \leq 2K \), a BAPA is efficient. Therefore, a BAPA is
feasible for the realization \((y_H, y_L)\) if there is a \(z\) that weakly improves the payoffs of \(H, L,\) and \(F\), i.e.,

\[ A1 \quad t_H z \geq t_H y_H - K, \]

\[ A2 \quad t_L (1 - z) \geq t_L y_L - K, \]

\[ A3 \quad t_H z + z = t_L (1 - z) \leq t_H y_H + t_L y_L. \]

A \(z\) exists that satisfies equations \([A1]–[A3]\) if and only if

\[ y_H - \frac{K}{t_H} \leq 1 - y_L + \frac{K}{t_L} \]

and

\[ (t_H - t_L) \left( y_H - \frac{K}{t_H} \right) \leq t_H y_H - t_L (1 - y_L). \]

These two conditions are satisfied if and only if \(x \equiv y_H + y_L - 1 \leq x^* = K(t_H + t_L) / (t_H t_L)\).

Now suppose that the players enter a BAPA if and only if \(x \leq x^*\). This implies that \(\hat{\mu} = E[x | x \geq x^*]\). Because \(x^* \leq \hat{\mu} \leq 1\), there exists a \(p \in [0,1]\) such that \(\hat{\mu} = E[x | x \geq x^*] = px^* + (1 - p)\). Substituting this expression for \(\hat{\mu}\) into the condition

\[ K \leq \frac{\hat{\mu} t_H}{t_H t_L} / (t_H + t_L) \]

and using the fact that \(x^* = K(t_H + t_L) / (t_H t_L)\) implies that \(K \leq \frac{\hat{\mu} t_H}{t_H t_L} / (t_H + t_L)\) is equivalent to \(K \leq t_H / t_H + t_L\). Therefore, it follows from Proposition 1(a) that the players play their heavy audit strategies in the absence of a BAPA for all \(K \leq t_H / (t_H + t_L)\).

(b) We show that if the players choose their light audit strategies in the reporting game, a BAPA is never feasible, and if the players never enter a BAPA, light auditing is the best response in the reporting game when \(K \geq \mu t_H / (t_H + t_L)\).

When the players would choose the light audit strategies in the reporting game, a BAPA is not efficient because \(C \geq K(t_H - t_L) / t_H t_L\).

If the players never enter a BAPA, \(\hat{\mu} = \mu\), so that \(K \geq \mu t_H / (t_H + t_L)\) is equivalent to \(K \geq \mu t_H / (t_H + t_L)\). Proposition 1(b) then implies that the players will choose their light audit strategies in the reporting game. QED

Proof of Proposition 4

(a) Identical to the proof of Proposition 3(a).

(b) We show that when the players choose their light audit strategies in the reporting game, and the off-equilibrium path belief is that \(\hat{\mu} = 1\), a BAPA is always feasible, and that the off-equilibrium path belief that \(\hat{\mu} = 1\) is the only belief that is consistent with the conjectured equilibrium. When the players would choose the light audit strategies in the reporting game, they incur audit costs of \(\delta, K = (t_H - t_L) / t_H\). Because \(C \leq K(t_H - t_L) / t_H\), a BAPA is feasible. Therefore, a BAPA is feasible if for the realization \((y_H, y_L)\) there is a \(z\) that weakly improves the expected payoffs of \(H, L,\) and \(F\). The payoffs of \(H, L,\) and \(F\) in the light audit equilibrium are

\[ t_L \left( 1 - y_L + \frac{K x}{\hat{\mu} t_H} \right), \]

where \(\hat{\mu} = 1\) given the off-equilibrium path beliefs of \(H\) and \(L\).

\[ t_H y_L - t_L (1 - y_L). \]

Therefore, the conditions under which a feasible BAPA exists are

\[ A4 \quad t_H z \geq t_H y_H - \frac{K x}{\hat{\mu} t_H} / \left( 1 - \frac{t_L}{t_H} \right) \]

\[ A5 \quad t_L (1 - z) \geq t_L \left[ 1 - y_L + \frac{K x}{\hat{\mu} t_H} \right], \]

\[ A6 \quad t_L z + t_L (1 - z) \leq t_L y_H + t_L (1 - y_L). \]

A \(z\) exists that satisfies equations \([A4]–[A6]\) if and only if

\[ y_H - \frac{K x}{\hat{\mu} t_H} / \left( 1 - \frac{t_L}{t_H} \right) \leq y_L - \frac{K x}{\hat{\mu} t_H}, \]

which is true for all \(x \leq 1\) because \(\hat{\mu} = 1\). Therefore, a BAPA is feasible for all \(x\).

It now remains to show that the off-equilibrium path belief that \(\hat{\mu} = 1\) is the only belief that is consistent with the conjectured equilibrium. For any other belief, a BAPA would not be feasible for realizations of \(x \geq \hat{\mu}\) and so the players would not enter a BAPA when \(x \geq \hat{\mu}\). However, the only solution to \(E[x | x \geq \hat{\mu}] = \hat{\mu}\) is \(\hat{\mu} = 1\). This implies that the condition \(K \geq \mu t_H / (t_H + t_L)\) is equivalent to \(K \geq t_H / (t_H + t_L)\). Therefore, a BAPA is feasible for all \(x\) if \(K \geq t_H / (t_H + t_L)\) and \(C \leq K(t_L - t_L) / t_H\). QED
Proof of Proposition 5

In the absence of a BAPA program, aggregate audit costs would be $2K$ if $K \leq \mu t_H t_L / (t_H + t_L)$, and $K(t_H - t_L) / t_H$ if $K \geq \mu t_H t_L / (t_H + t_L)$ (Proposition 1 with \(\hat{\mu} = \mu\)). BAPA costs are always $C$.

(a) From Propositions 3(a) and 4(a), it follows that a BAPA will occur when $x \leq x^*$ and the players will choose their heavy audit strategies when $x > x^*$. In the absence of the BAPA program, the players would choose their heavy audit strategies for all $x$ because $K \leq \mu t_H t_L / (t_H + t_L)$. Because $C \leq 2K$, the existence of the BAPA program decreases compliance costs when $x \leq x^*$ and has no effect on compliance costs when $x > x^*$.

(b) From Proposition 4(b), it follows that a BAPA will occur for all $x$. In the absence of the BAPA program, the players would choose their light audit strategies for all $x$. Because $C \leq K(t_H - t_L) / t_H$, the existence of the BAPA program decreases compliance costs for all $x$.

(c) From Proposition 3(a), it is an equilibrium for a BAPA to occur when $x \leq x^*$ and for the players to choose their heavy audit strategies when $x > x^*$. In the absence of the BAPA program, the players would choose their light audit strategies for all $x$ because $K \geq \mu t_H t_L / (t_H + t_L)$. Because $C \geq K(t_H - t_L) / t_H$, the existence of the BAPA program increases compliance costs for all $x$ when that equilibrium occurs. However, from Proposition 3(b) it is also an equilibrium for the players to not enter into a BAPA and choose their light audit strategies. In that case, compliance costs would not change.

(d) From Proposition 4(a), it follows that a BAPA will occur when $x \leq x^*$ and the players will choose their heavy audit strategies when $x > x^*$ because $K \leq \mu t_H t_L / (t_H + t_L)$. In the absence of the BAPA program, the players would choose their light audit strategies for all $x$ because $K \geq \mu t_H t_L / (t_H + t_L)$. Because $C \leq K(t_H - t_L) / t_H$, the existence of the BAPA program decreases compliance costs when $x \leq x^*$ and increases compliance costs when $x \geq x^*$. QED