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MEASURING THE PERFORMANCE OF THE PROPERTY TAX ASSESSMENT PROCESS

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ABSTRACT - *Most studies of property tax assessment performance focus on one-dimensional measures of assessment performance, such as the coefficient of dispersion. In this paper, the efficiency of the assessment process is analyzed incorporating both measures of output or performance and measures of resource cost. First, the coefficient of variation is adjusted to account for differences in the degree of difficulty of the assessing environment. Then a measure of efficiency in the assessment process is derived and implemented for assessment jurisdictions in the state of Illinois. The paper concludes with a discussion of the use of measures of output and efficiency in ranking assessment performance.*

INTRODUCTION

In the existing literature, the performance of the assessment process for the property tax is often evaluated solely in terms of degree of uniformity that is achieved. Assessment equity or uniformity of the property tax system is gauged with the help of the coefficient of dispersion (CD)[1] or similar measures of uniformity. The CD is a measure of the degree of nonuniformity in the ratio of assessed value (established by the assessment process) to market value (established by sales studies) of properties within a jurisdiction. The higher the value of the coefficient, the lower is the quality of performance of the assessment process.[2]

Assessment uniformity (as measured by the CD), however, addresses only one of several dimensions of property tax assessment. In an overall evaluation of assessment performance, such a measure should be combined with information on the resources used in the assessment process. A low CD achieved at a high cost is not necessarily a sign of an efficient tax system. For instance, in a study of property tax assessment in the state of Illinois, 66 percent of the counties whose CDs were lower than the median level had tax assessment costs (budget expenditures including salary) per parcel higher than the average cost of tax assessment for

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the state. In a few such counties, the cost of tax assessment per parcel was very high.[3] These observations suggest that the concept of efficiency in property tax assessment needs to be redefined and broadened to include measures of both uniformity and cost.

The present paper addresses the issue of efficiency in the assessment process by integrating measures of uniformity (output or performance) with measures of the cost of the resources used in the process. In this study, measures of uniformity are appropriately adjusted for the difficulty of the environment that assessors face. Then, a well-known technique used for evaluating efficiency in the private sector is adapted to derive measures of allocative and technical efficiency in assessment for counties in Illinois. The paper concludes with a discussion of the trade-off between uniformity and assessment cost.

THE CONCEPT OF EFFICIENT PROPERTY TAX ASSESSMENT

To achieve efficiency, assessors should value parcels subject to the property tax as close to their market value (or some alternative legal objective) as possible for a given budget. Operationally, the assessor should minimize the CD for a given expenditure of resources. (Alternatively, the assessor should minimize cost for a given CD.) In this process, the assessor must allocate available resources among various assessment tools. Assessment performance depends on the quality and quantity of the personnel used as well as other resources used in the process, *i.e.,* the organization's production function. Increases in the quality and quantity of personnel and other resources should result in better performance as reflected by a lower level of the CD. Thus, the performance or output (measured in terms of the inverse of the CD) is expected to be positively related to the cost of tax administration for a given assessment environment. This relationship between output and the cost of the property tax assessment system is analogous to that of a cost function for a firm in the private sector. An efficient property tax organization, therefore, is one that incurs the minimum expenditure for a given amount of output (1/CD).

A higher than minimum cost of tax assessment (COTA) for a given output results from two effects. First, the condition required for cost minimizing operation may not be fulfilled. This condition demands equality between the ratio of the marginal products (MP) of the inputs and the ratio of their prices (P), *i.e.,* $MP_i/MP_j = P_i/P_j$ where the subscripts *i* and *j* denote the two inputs. The absence of this equality indicates that inputs are employed in inefficient proportions leading to the higher operating costs. In the literature, this type of problem is known as *allocative inefficiency*. In the presence of such an inefficiency, it is possible to reduce cost without reducing output by substituting one factor for another.

Another source of inefficiency occurs when an organization (which may be allocative efficient) uses more of total units of inputs than other similar organizations producing the same output. This is known as *technical inefficiency.* Since such inefficiency results from the excessive use of inputs, it is possible to reduce the volume of inputs (and thus total cost) without reducing the level of output. Both concepts are shown in Figure 1.

In Figure 1, the horizontal and vertical axes measure inputs *x* and *y* as a proportion of output. I_{01} and I_{02} are two homogenous unit isoquants (represent-

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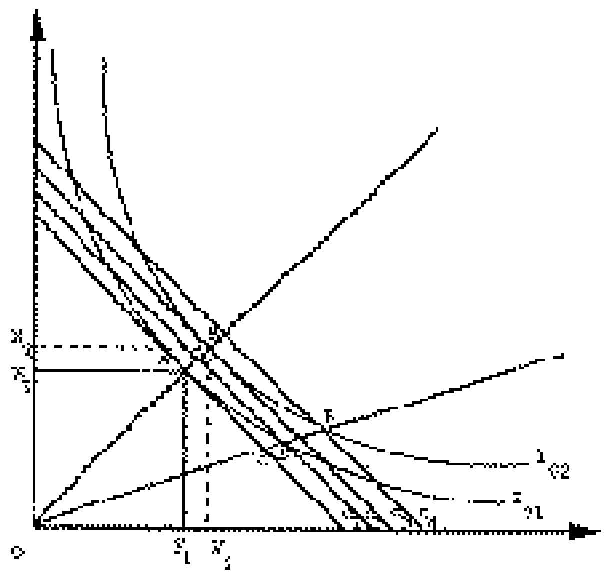
FIGURE 1. Technical and Allocative Efficiency



ing the same level of output, which in this case is assessment uniformity).[4] The slope of the isoquant is the ratio of the marginal products of two inputs. Four isocost lines, C_1, C_2, C_3, and C_4, indicate four different levels of cost. The slope of the isocost curve is the ratio of the input prices. In this example, the inputs might represent the number of assessors and computers. Output levels might be performance levels for assessment (1/CD) for a given size jurisdiction where size is measured by the number and composition of parcels.

The allocative efficient organization is the one that uses the input mix that minimizes the cost associated with a given level of output. Obviously, in the diagram, this is point A, where the isoquant curve and isocost line relating the minimum feasible cost C_1 are tangent. The organization operating at point D on isoquant I_{01} is allocative inefficient, because the cost for a given level of output at this point is more than the minimum feasible cost. There is a potential for reducing cost by substituting input *x* for *y*. In the diagram, this type of inefficiency can be defined as CD/OD (which is also (1 – OC/OD) or (1 – TC_1/TC_2)), where TC represents total cost relating to an isocost curve). Note that the organization operating at point B is allocative efficient, but it is technically inefficient as it uses more of both inputs for the same volume of output. The technical inefficiency at this point can be defined as (1 – OA/OB). The organization operating at point E is both allocatively and technically inefficient. At this point of operation (where total cost is TC_4), the additional cost resulting from the inefficiency is ($TC_4 – TC_1$), which is the combined effect of the technical inefficiency, measured by ($TC_4 – TC_2$), and the allocative inefficiency, measured by (TC_2

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– TC_1). The overall inefficiency is defined as the ratio (1 – TC_1/TC_4), which is (1 – OC/OE). This ratio measures the proportion by which cost exceeds the minimum feasible cost.

In the empirical analysis that follows, the overall inefficiency in the assessment process, which is the focus of this paper,[5] is measured as one minus the ratio of the minimum feasible cost (TC_1) to the actual tax assessment cost. Data concerning the cost and quality of actual tax assessments are available. However, a problem arises in the determination of minimum cost, which is to serve as the efficiency benchmark, in an environment where the factors affecting assessment cost vary from county to county. This problem is addressed by adjusting the CD to account for factors influencing assessment performance that are outside the control of tax administrations before the estimation of a cost frontier.[6]

ESTIMATION OF COST FRONTIER

The cost frontier is defined as the set of minimum feasible assessment costs (COTA), where the minimum cost of assessment is a function of jurisdiction-specific factors that are outside the control of assessors. The actual observed COTA points lie on or above this frontier. The difference between the observed (C) and the minimum feasible COTA is one-sided only. The cost function to be estimated can be expressed as follows:

1

$$C_i = \alpha + \Sigma\beta_j X_{ij} + e_i; \qquad i = 1, \ldots, P$$

where $e_i \geq 0$. This specification assumes that the difference between the observed COTA and minimum feasible COTA is generated only by inefficiency. Subscript *i* denotes the *i*th property tax unit, with *P* being the total number of assessing units.

Since equation 1 assumes that *e* is a one-sided disturbance, the usual assumption of regression analysis that $E(e) = 0$ does not apply. Thus, the ordinary least-squares (OLS) method cannot be used to estimate this equation. In its place, a method known as corrected ordinary least-squares (COLS) is employed to estimate the cost frontier (Farrell, 1957). This approach is a modified form of OLS. The COLS approach assumes that e_i has a one-sided distribution with $E(e) = k$ (with k not equal to zero). Equation 1, after adding and subtracting k, can be written in the following form:

2

$$C_i = \alpha + k + \Sigma\beta_j X_{ij} + e_i - k.$$

The new disturbance term, $u_i = e_i - k$, satisfies all the usual conditions for OLS since $E(u_i) = 0$. The OLS technique can therefore be employed to estimate the best linear unbiased estimate of β_j except that OLS will produce an estimate for the constant term of $\alpha + k$. The simplest way to estimate the latter is to shift it down until no residual is negative and one is zero.[7] Let α^{OLS} (which is $\alpha + k$) be the OLS estimate of the intercept of equation 2 and min (u_i) be the value of the smallest of the residuals. In this situation, the COLS estimate of α (α^{COLS}) is given by (α^{OLS} – min (u_i)). It can be shown that α^{COLS} is the consistent estimate of α. Inefficiency can then be estimated from the corrected residuals which are defined as $u^{COLS} = u^{OLS} - \min(u_i)$.[8] The u^{COLS} will be zero for the most efficient tax jurisdictions. This procedure amounts to

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establishing the lowest cost organization (appropriately normalized for the difficulty of the assessing task) as the norm for efficiency.

To estimate equation 2, a statistical cost function[9] is employed. Its arguments are the output of the property tax assessment process and the work load. The work load is measured by the number of parcels to be assessed and by the composition of the parcels, since the difficulty in assessment varies from one kind of parcel to another. Let β_j be the average cost relating to the *j*th type of parcels and *RES*, *BUS,* and *FAR* be the number of residential, business, and agricultural properties, respectively. Then the statistical cost function can be expressed as follows:

3a

$$COTA = \beta_4(1/CD) + \beta_1 RES + \beta_2 FAR + \beta_3 BUS.$$

It has been shown that the environment in which the assessor works influences the assessment process (Chicoine and Giertz, 1986). For instance, a higher level of education of the taxpayers, which promotes awareness of assessment inequities and generates appeals, may be expected to help reduce the magnitude of CD, although it may not have any direct impact on the cost of assessment. Similarly, a large percentage of new housing units in a jurisdiction helps the assessor to get good information about housing prices and is expected to be positively related to the output level (1/CD) of the organization. High tax levels are also expected to have a positive impact on the output level by encouraging greater taxpayer awareness. On the other hand, heterogeneity in housing stock is expected to produce a negative impact on the output level.[10]

The effects of these factors are beyond the control of the assessors. Thus, in order to know the actual level of assessor performance (which is an important step in measuring efficiency), the impact of these noncontrollable factors must be adjusted for in determining a normalized output level. A plausible way of dealing with such effects is to estimate an equation relating the unadjusted output level and the environmental factors. The results of this estimation are then used to compute an adjusted CD. The CD thus obtained may be considered as the normalized CD and is used in equation 3. The following regression equation[11] (based on (Chicoine and Giertz, 1986, 1990) is used to obtain the normalized value of the CD:

4

$$\ln (1/CD) = \alpha_0 + \alpha_1 CHV + \alpha_2 CHN + \alpha_3 HET + \alpha_4 ATB + \alpha_5 ED + \alpha_6 NJ + \alpha_7 CNTYPE + e.$$

Equation 3a, using the normalized value of the CD discussed above, is estimated in a loglinear form. The choice between linear and loglinear form is made on the basis on the widely used PE test (Kmenta, 1990).[12] The final form of equation 3a is as follows:

3b

$$\ln COTA = \beta_0 + \beta_4 \ln (1/\widehat{CD}) + \beta_1 \ln RES + \beta_2 \ln FAR + \beta_3 \ln BUS + \beta_5 CNTYPE + u.$$

In the above equation, one additional variable, *CNTYPE,* is also included. This is a dummy variable employed to

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capture the effects of the legal structure of counties in Illinois on the cost of tax assessment. This deals with the question of whether the administrative structure of the property tax system in commission counties differs from that in counties with townships. The variable is 0 for the commission county and 1 for the township county.

Data on *CD, RES, FAR, BUSS,* and COTA pertaining to 1983 for 88 counties are available from a survey conducted in Illinois (Chicoine and Giertz, 1986). The data on *RES* also include farm houses used for residences.

A comparison of the observed COTA and the COTA estimated from equation 3a provides a measure of the level of efficiency in a particular county. The expression 100 * exp $[-(u_i - \min(u_i))]$ measures the relative efficiency of a county in comparison to the county with the most efficient property tax administration in Illinois.[13] This efficiency measure is the ratio of the minimum cost to the actual cost of assessment, appropriately adjusted for the specific characteristics of a county. For example, a value of 100 indicates that a county's assessment cost is equal to that of the minimum cost county; a value of 50 shows that a county's cost is double that achieved in the most efficient county.

TABLE 1
REGRESSION RESULTS

Explanatory Variables	Equation 4[a]	Equation 3b[a]
CHV	–0.0025 (–3.0139)	—
CHN	0.0122 (3.5874)	—
HET	–0.0127 (–2.1621)	—
ATB	0.0004 (2.5205)	—
ED	0.0085 (1.6422)	—
NJ	–0.0151 (–2.0706)	—
CNTYPE	0.2185 (2.2838)	–0.1917 (–1.6361)
Ln (1/CD)[b]	—	0.4796 (2.3253)
Ln *RES*	—	0.2109 (1.4212)
Ln *BUS*	—	0.3026 (2.2726)
Ln *FAR*	—	0.0672 (0.6405)
CONSTANT	–4.1204	9.1317[c]
$\bar{R}^2$	0.55	0.70
F	16.592	40.477
DW	1.8559	2.346

[a]Figures in parentheses are *t* values.
[b]Ln is natural log.
[c]Constant term in natural log.

THE ESTIMATION OF EFFICIENCY

The results of the estimation of equations 3b and 4 are presented in Table 1. In both equations, all variables are significant at reasonable significance levels (five and ten percent), except the coefficient relating to farm land. The Glesjer test shows the presence of heteroskedasticity. The appropriate correction (using White's heteroskedastic consistent covarinace) improves the significance level of the coefficients.[14] In addition, when the RESET test is carried out, the test does not suggest the omission of any relevant variables in the model.

The signs of the coefficients are as expected. The positive signs of the four variables (ln (1/CD), ln *RES*, ln *BUS*, and ln *FAR*) in the estimated cost equation 3b indicate that the cost of tax assess-

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ment is positively related to the quality of assessment and various measures of the workload represented by the number of various types of property. The low and insignificant value of the factor related to agricultural properties (ln *FAR*) suggests that its effect on the cost of property tax assessment is not significant, while business and residential properties play the major role in determining assessment cost.[15] Since the coefficients are elasticities, comparisons of coefficients can be made directly on the basis of their magnitudes. The analysis indicates that output itself (as measured by assessment uniformity, ln (1/CD)) is the most important factor, with business properties, and residential properties, respectively, second and third in importance.

It should be noted that the cost function is close to homogenous of degree 1 in output, *RES, BUS,* and *FAR,* with the sum of β_1, β_2, β_3, and β_4 equal to 1.05.[16] Two implications emerge from this result. First, if the output level (*i.e.,* assessment quality) and the different types of parcels are all increased by a constant percentage, then the total cost of tax assessment will also increase by the same percentage. Second, if the output level is held constant and with $\beta_2 + \beta_3 + \beta_4 < 1$ (actually 0.85), an increase in the number of all parcels by a given percentage will result in a smaller percentage increase in costs or a reduction in the average per parcel cost of assessment. In effect, there are economies of scale in administration.

The negative and significant value of the *CNTYPE* indicates that the cost frontier for the township counties differs from that for the commission counties or that commission counties are systematically less efficient than township counties.[17] Equation 3a can be used to measure the size of the cost bias effect of the commission counties *versus* the township counties for a given level of control factors (used as determinants for the cost function). Dividing the cost function for township counties ($COTA^t$) by that for commission counties ($COTA^c$) gives: $\ln COTA^t / \ln COTA^c = \beta_5$ or $COTA^c = \exp(-\beta_5) COTA^t$. Since $\beta_5 = -0.1912$, $COTA^c = 1.21 COTA^t$.

In words, this equation states that, for a given set of characteristics, commission counties are expected to have 21 percent higher costs than township counties for the same CD level. The efficiency is estimated as follows:

5

$$\text{efficiency} = 100 * \exp[-((\ln COTA - \ln COTA) - (-0.8897))]$$

where

$$\ln COTA - \ln COTA = u \text{ and}$$

$$-0.8897 = \min(u).$$

The estimates of efficiency are presented in Table 2. Column (3) presents county rankings based only on the level of adjusted output for each county and ignoring assessment costs. Column (4) gives the efficiency levels as calculated from equation 5, while column (5) gives the ranking of counties according to this measure. Finally, column (6) gives the per parcel cost of tax assessment.

The average efficiency measure for all 88 counties is 44 percent.[18] The results show that the rankings based on the cost of tax assessment and output level (column (5)) differ greatly from the rankings based only on the level of output (column (3)).[19] The Spearman's

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TABLE 2
RANKING OF COUNTIES

Number (1)	County (2)	Rank Based on Adjusted Output[a] (3)	Efficiency Level in the Present Study (%) (4)	Rank Based on Column 4 (5)	PPCOTA[b] ($) (6)
1	Rock Island	54	100	1	2.04
2	Cass	74	92	2	4.20
3	Williamson (C[c])	48	85	3	3.02
4	JoDaviess	84	82	4	3.69
5	Macoupin	83	78	5	2.34
6	Gallatin	61	78	5	3.62
7	White	7	76	7	4.08
8	Sangamon	68	73	8	2.64
9	Vermilion	25	71	9	2.64
10	Kankakee	28	70	10	3.23
11	Tazewell	81	70	10	3.15
12	Hancock	75	65	12	4.40
13	Whiteside	53	65	12	3.89
14	Warren	46	62	14	5.41
15	Calhoun (C)	30	62	14	8.54
16	Putnam	72	61	16	7.51
17	Dewitt	27	61	16	6.63
18	Moultrie	35	58	18	7.65
19	Clay	18	56	19	4.72
20	Adams	4	55	20	4.65
21	Jackson	86	53	21	6.36
22	Will	45	53	21	3.27
23	Champaign	29	53	21	5.45
24	Montgomery	70	52	24	4.08
25	Henry	60	49	25	5.31
26	Crawford	39	49	25	7.06
27	Scott (C)	69	48	27	9.26
28	Effingham	22	48	27	6.95
29	Boone	71	48	27	8.47
30	Stark	55	48	27	9.64
31	Menard (C)	66	47	31	8.60
32	Wabash (C)	14	47	31	10.12
33	Hamilton	38	47	31	6.04
34	Marion	51	47	31	5.12
35	Edgar (C)	15	46	35	7.35
36	Fayette	41	45	36	5.32
37	Bond	24	45	36	7.14
38	Jersey	73	45	36	8.61
39	Alexander (C)	85	44	39	11.16
40	Logan	17	42	40	8.23
41	Pike	8	42	40	5.98
42	Saline	2	42	40	6.93
43	Richland	12	42	40	9.57
44	Winnebago	40	41	44	4.89
45	Cumberland	79	41	44	7.40
46	Schuyler	82	41	46	8.19
47	Hardin (C)	33	39	47	14.26
48	DeKalb	9	39	47	9.71
49	Henderson	13	38	49	10.35
50	Pulaski (C)	88	37	50	10.45
51	Piatt	31	37	50	9.56
52	Lawrence	57	36	52	7.77
53	Woodford	56	35	53	10.93
54	Morgan (C)	44	35	53	11.01
55	Shelby	62	35	53	7.58
56	Franklin	77	35	53	7.08
57	Washington	42	35	53	6.79
58	Union (C)	78	34	58	9.70
59	Ogle	76	34	58	8.59

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TABLE 2 (Continued)

Number (1)	County (2)	Rank Based on Adjusted Output[a] (3)	Efficiency Level in the Present Study (%) (4)	Rank Based on Column 4 (5)	PPCOTA[b] ($) (6)
60	Lee	52	34	58	7.72
61	Randolf (C)	43	34	58	11.24
62	Perry (C)	65	34	58	9.79
63	Brown	19	33	63	11.88
64	Ford	58	33	63	14.41
65	Iroquis	87	33	63	8.13
66	Clinton	26	32	66	8.64
67	McHenry	20	32	66	6.84
68	Mercer	5	31	68	12.59
69	Christian	21	31	68	8.35
70	Lake	34	30	70	5.18
71	Carroll	80	29	71	10.52
72	Pope (C)	6	29	71	7.13
73	Knox	49	28	73	9.38
74	Jasper	16	27	74	10.03
75	McLean	11	27	74	9.93
76	Macon	37	26	76	7.78
77	Livingston	47	26	76	10.58
78	Edgar	10	26	76	10.63
79	Kane	3	25	79	7.55
80	Greene	63	25	79	14.57
81	Monroe (C)	1	24	81	16.87
82	Peoria	64	24	81	7.86
83	Clark	59	23	83	13.92
84	Fulton	67	23	83	11.84
85	Stephenson	32	22	85	14.00
86	Bureau	50	22	85	13.63
87	Madison	23	21	87	5.95
88	Grundy	36	14	88	26.17

[a]Obtained from column 4 in Table 5.4 in Chicoine and Giertz (1986); the ranking number is slightly varied since 88 counties are considered in this paper, while the Chicoine and Giertz study includes 102 counties.
[b]PPCOTA = per parcel cost of tax assessment (average = $6.02).
[c]C = commission county.

correlation coefficient between the two rankings is 0.04, indicating virtually no relation between them. For example, compare Sangamon County with Fulton County. When ranked on the basis of adjusted output alone (from column (3)), the two counties are nearly identical, ranking 68th and 67th, respectively. However, when costs are considered as well as output, Sangamon County, with a 73 percent efficiency rating, ranks 8, while Fulton County, with a 23 percent rating, ranks 84.

The relationship between the level of efficiency and the size of counties (as measured by the number of parcels) is examined next. A quadratic relationship is hypothesized between the number of parcels (TP) and efficiency (EFF). This relationship is estimated in equation 6.

6

$$EFF = 40.172 + 0.2777 * 10^{-3}\,TP - 0.2041 * 10^{-8}\,TP^2; \quad R^2 = 0.033$$
$$(1.5705) \qquad (-1.6933)$$

Both regression coefficients are significant at the 10 percent level, with the latter being significant at the five percent level as well (one-tail test). The

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results show that efficiency increases up to about a 68,000 parcel-size county and then starts to decline.[20] The results in equation are only suggestive, so strong policy recommendations advocating increased district size (through, for example, the consolidation of counties) are problematic.[21] However, the results suggest that an optimum relationship between efficiency and district size may exist in property tax administration. This relationship should be examined further with more detailed data to determine if efficiencies could be achieved through consolidation.

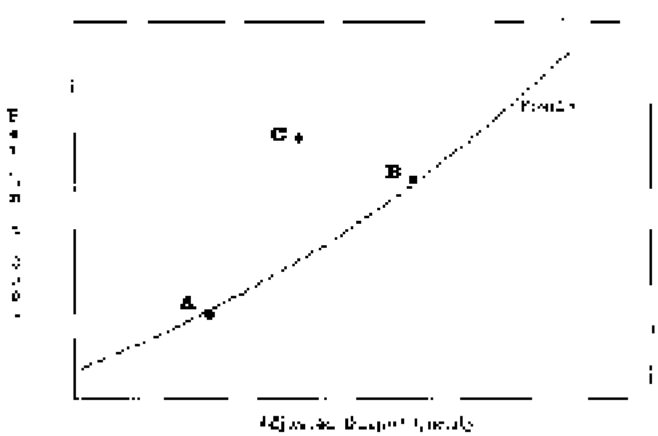
Conclusions

The analysis presented here suggests that the evaluation of overall assessment performance is more complex than simply comparing measures, such as the coefficient of dispersion, to rank assessors. As noted previously, it is first necessary to adjust the performance of assessors for the degree of difficulty of the assessing task. After this is accomplished, it is then necessary to weigh both the adjusted output measure and the cost of assessment to reach a final conclusion.

Consider the cost frontier displayed in Figure 2, which is similar to the one developed in the preceding section. Points A, B, and C represent three different assessing outcomes, where A and B are on the frontier (*i.e.,* are efficient) and C is inefficient. Using a one-dimensional measure of performance, such as the coefficient of dispersion, outcome B dominates C which in turn is superior to A. However, when costs are considered as well as output, B remains superior to C in that B represents a higher quality of assessment achieved at a lower cost, but it is impossible to rank unambiguously B *versus* A or A *versus* C.

To make these sorts of comparisons, it is necessary to know the value that is placed on improved assessing performance and then to compare it to the extra cost involved. In comparing A and B, it is quite likely that A would be

FIGURE 2. Cost and Output Quality Relationship



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preferred to B in an environment where property tax rates are very low. With low property tax rates, the consequences of nonuniform assessments are less severe than in a high tax environment. In this situation, choosing a lower quality of assessment at a lower cost is a quite rational decision.

This suggests that it may be impossible to have precise rankings of assessor performance when both cost and output are considered. Efficiency measurements alone are useful, however, in that they provide guidance as to whether assessors are using their available resources in an effective manner.

ENDNOTES

We wish to thank two anonymous referees and the editor for helpful suggestions on an earlier draft of this paper.

[1] The coefficient of dispersion is given by the following formula: $(\sum|a_i/s_i - (a/s)_{med}|)/n(a/s)_{med}$. It is the average deviation of the actual sales assessment ratios (*i.e.,* the ratio of assessed value (*a*) to actual selling price (*s*) as measured by sales-assessment studies) from the median ratio $(a/s)_{med}$ divided by the overall median ratio.

[2] Since assessing performance is affected by the environment in which assessors work, comparisons among jurisdictions is sometimes difficult. To make interdistrict comparisons more meaningful, actual CDs can be adjusted by taking into account factors influencing the difficulty of the assessment environment. See Chicoine and Giertz (1986, 1990).

[3] For instance, in Grundy County, whose CD was 25 percent lower than the median level CD in the state, the cost of property tax asses~ment per parcel was four times higher than average cost for the state. See Grundy county in Table 2.

[4] In this diagram, unlike usual indifference curves, isoquant I_{02} does not represent a higher level of output as compared with I_{01} even though it is farther from the origin. Both isoquants represent the same level of output.

[5] Although it is useful to have the estimates of technical and allocative efficiency separately, it is difficult to disaggregate overall efficiency, because the estimation of allocative inefficiency separately requires data on prices for all kinds of inputs including their movements over time. Such data are generally not available. Even if they were available, a model incorporating all such information would be complex and difficult to estimate.

[6] See Chicoine and Giertz (1990). They develop a technique for normalizing assessment uniformity measures by adjusting for the difficulty of the assessment task.

[7] Alternatively, α may be estimated by the method suggested by Richmond. He has shown that, if observations on the error term *e* have a gamma distribution, the variance of OLS residuals is given by var $(u) = k$. α^{COLS} is obtained by using the estimate of var (*u*) instead of shifting α^{OLS} down until no residual is negative. This can be shown as follows:

$$C_i = \alpha^{OLS} - \text{var}(u) + \Sigma\beta_j X_{ij}.$$

A difficulty with this kind of COLS technique is that, even after correcting the constant term, some of the residuals may still have the negative sign so that some COTA observations end up below the cost frontier. This makes this technique a somewhat awkward basis for computing the inefficiency of individual observations (Richmond, 1974).

Another difficulty pointed out by Forsund, Lovell, and Schmidt (1980) is that the correction to the constant term is not independent of the distribution assumed for e_i. If one assumes a gamma distribution of the observations on e_i, then the constant term is corrected by the variance estimate of OLS residuals. If the distribution is assumed to be exponential, then the square root of the variance estimate provides the correction to the constant term. Thus, the gamma distribution and exponential distribution yield systematically different corrections for the constant term.

These two problems do not arise in the case of the COLS technique used in the present paper.

[8] This expression, when subtracted from one, is similar to the inefficiency measure $(1 - TC_1/TC_4)$ developed in the preceding section.

[9] In the literature, such statistical cost frontier functions are widely used to analyze the hospital sector and educational institutions. The arguments in such functions are the factors affecting the cost of the organizations in question. Examples of such studies include Wagstaff (1989), Hammond (1986), and Levitt and Joyce (1987). The first study estimated the efficiency in Britain's hospital sector, the second did so for British crematoria, and the third dealt with a variety of public sector activities. The statistical cost frontier function pertains to a family of different techniques for estimating the frontier function. Recently, the frontier function's application has widened to include a number of public sector activities. Interested readers may

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consult the following references that deal with estimation techniques: Baur (1990), Green (1990), Green (1980), Schmidt and Lovell (1979), and Schmidt and Sickles (1984). A number of applications are addressed in the following references: Feldstein (1967), Barrow (1988), Cowing, Holtman and Powers (1990), Cubbin, Domberger and Meadowcroft (1987), Domberger, Meadowcroft and Thomspon (1980), McGuire (1987), Jesson, Mayston and Smith (1987), Mayston and Smith (1987), Lave and Lave (1970), and Steele and Gray (1982).

[10] For detail about the effects of environmental factors on CD, see Chicoine and Giertz (1989).

[11] The form of the equation is similar to one used in Chicoine and Giertz (1986, 1990). The variables in equation 4 are as follows: *CHV* = changes in housing value, *CHN* = changes in number of houses, *HET* = heterogeneity in housing stock (age), *ATB* = average tax bill–tax bill for median value house, *ED* = percentage of population 25 and over with 12 years of education, *NJ* = number of assessing jurisdiction, and *CNTYPE* takes 0 for a commission county and 1 for a township county. The adjusted measure of performance is the difference between the estimated COD (from equation 4) and the actual COD.

[12] The PE test favors a loglinear form if the coefficient of $r1$ is significant in the linear equation and the coefficient of $r2$ is insignificant in the loglinear equation, where $r1$ = ln [(estimated COTA) – (estimated ln COTA)] and $r2$ = [(estimated COTA) – exp (estimated ln COTA)]. $r1$ is an argument in the linear form equation and $r2$ is a argument in the loglinear equation. For the equation in question here, the t value of the coefficient of $r1$ = –2.2744 and that of $r2$ = 1.2029. The latter is insignificant and the former is significant at a 2.5 level of significance. For details, see Kmenta (1990).

[13] In the empirical analysis, measures of efficiency (in percentage terms) are used rather than the measures of inefficiency developed earlier. 100 minus this measure of efficiency yields a measure of inefficiency (again, in percentage terms).

[14] The multiple correlations of *RES* (91) and *BUSS* (91.5) on other independent variables are very high, indicating the possible presence of multicollinearity problems. However, values of the condition number (less than 6) derived with the help of the eigenvalues of $(X'X)$ do not indicate that a severe problem exists. $(X'X)$ is the moment matrix of the explanatory variables of equation 3a or 3b.

[15] This is understandable since farm land in Illinois is assessed under a use-value approach. Its value is determined by a formula based on soil productivity, crop prices, and factor prices and is not determined by local assessors.

[16] Test value = 0.0603, standard error = 0.2041, t statistic = 0.2954 with 82 degrees of freedom, and $F_{1,82}$ = 0.0873. Both the t and F statistics indicate that the sum of the regression coefficients (1.05) is not significantly different from one.

[17] The cost frontier for township counties is

$$\ln COTA = 8.9406 + 0.4796 \ln CD + 0.2109 \ln RES + 0.3026 \ln BUSS + 0.0672 \ln FAR$$

and that for commission counties is

$$\ln COTA = 9.1317 + 0.4796 \ln CD + 0.2109 \ln RES + 0.3026 \ln BUSS + 0.0672 \ln FAR.$$

[18] This is a simple average: the sum of the efficiency values for the counties divided by the number of counties.

[19] Only 88 of the 102 counties in Illinois were used because of data limitations.

[20] The optimum size of a county where efficiency is at a maximum can be found by the simple optimization technique. The result thus obtained shows that the most efficient property tax administration district is one dealing with 68,030 number of parcels.

[21] For example, increasing the number of parcels in rural units by consolidation may result in jurisdictions covering broad areas. These extensive jurisdictions may be more costly to assess compared to more compact urban jurisdictions of the same parcel size.

REFERENCES

Barrow, Michael. "Measuring the Performance of Local Education Authorities: A Frontier Approach." University of Sussex Economics Discussion Paper No. 88/1. Sussex: University of Sussex, 1988.

Barrow, Michael, and Adam Wagstatff. "Efficiency Measurement in the Public Sector: An Appraisal." *Fiscal Studies* 10 (February, 1989): 72–97.

Baur, Paul W. "Recent Developments in Econometric Estimation of Frontiers." *Journal of Econometrics* 46 (October, 1990): 39–56.

Chicoine, David L., and J. Fred Giertz. *Property Tax Assessment in Illinois: Structure and Performance*. Springfield, IL: Illinois Tax Foundation, 1986.

Chicoine, David L., and J. Fred Giertz. "Measuring the Performance of Assessors: Adjusting for the Difficulty of the Valuation Task." *Public Budgeting and Financial Management* No. 2 (1990): 177–203.

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Cowing, Thomas G., Alphonse G. Holtman, and S. Powers. "Hospital Cost Analysis: A Survey and Evaluation of Recent Studies." In *Advances in Health Economics and Health Services Research*, Vol. 4, edited by R. M. Scheffler and L. F. Rossiter. Greenwich, CT: JAI Press, 1987.

Cubbin, John, Simon Domberger, and Shirley Meadowcroft. "Competitive Tendering and Refuse Collection: Identifying the Sources of Efficiency Gains." *Fiscal Studies* 8 (August, 1987): 49–58.

Domberger, Simon, Shirley Meadowcroft, and David Thompson. "Competitive Tendering and Efficiency: The Case of Refuse Collection." *Fiscal Studies* 7 (November, 1986): 69–87.

Farrell, Michael J. "The Measurement of Productive Efficiency." *Journal of the Royal Statistical Society* 120 (1957): 253–81.

Feldstein, Martin S. *Economic Analysis for Health Service Efficiency: Econometric Studies of the British National Health Services*. Amsterdam: North-Holland, 1967.

Forsund, Finn R., C. A. Knox Lovell, and Peter Schmidt. "A Survey of Frontier Production Functions and of their Relationship to Efficiency Measurement." *Journal of Econometrics* 13 (May, 1980): 5–25.

Greene, William H. "A Gamma Distributed Stochastic Frontier Model." *Journal of Econometrics* 46 (October–November, 1990): 141–63.

Greene, William H. "Maximum Likelihood Estimation of Econometric Frontier Functions." *Journal of Econometrics* 13 (February, 1980): 27–56.

Hammond, Christopher J. "Estimating the Statistical Cost Curve: An Application of the Stochastic Frontier Technique." *Applied Economics* 18 (September, 1986): 971–84.

Jesson, David, David Mayston, and Peter Smith. "Performance Assessment in the Education Sector: Educational and Economic Perspectives." *Oxford Review of Education* 13 (1987): 249–66.

Kmenta, Jan. *Elements of Econometrics*, p. 521. New York: Macmillan Publishing Company, 1990.

Lave, Judith R., and Lester B. Lave. "Economic Analysis for Health Service Efficiency: A Review Article." *Applied Economics* 1 (1970): 293–305.

Levitt, M. S., and M. A. S. Joyce. *The Growth and Efficiency of Public Spending*. Cambridge: Cambridge University Press, 1987.

Mayshar, Joram. "Taxation with Costly Administration." *Scandinavian Journal of Economics* 93 No. 1 (1991): 75–88.

Mayston, David, and P. Smith. "Measuring Efficiency in the Public Sector." *Omega International Journal of Management Science* 15 (1987): 181–9.

McGuire, A. "The Measurement of Hospital Efficiency." *Social Science and Medicine* 24 (1987): 719–24.

Richmond, J. "Estimating the Efficiency of Production." *International Economic Review* 15 (June, 1974): 515–21.

Schmidt, Peter, and C. A. Knox Lovell. "Estimating Technical and Allocative Inefficiency Relative to Stochastic Production and Cost Frontiers." *Journal of Econometrics* 9 (February, 1979): 343–66.

Schmidt, Peter, and Robin C. Sickles. "Production Frontiers and Panel Data." *Journal of Business and Economic Statistics* 2 (October, 1984): 367–74.

Steele, R., and A. M. Gray. "Statistical Cost Analysis: The Hospital Case." *Applied Economics* 14 (October, 1982): 491–502.

Wagstaff, Adam. "Estimating Efficiency in the Hospital Sector: A Comparison of Three Statistical Cost Frontier Models." *Applied Economics* 21 (May, 1989): 659–72.