DEMAND FOR LOTTERY GAMBLING: EVALUATING PRICE SENSITIVITY WITHIN A PORTFOLIO OF LOTTERY GAMES

Michael A. Trousdale and Richard A. Dunn

This article introduces a new approach to analyzing whether lottery games are complements or substitutes, and whether a portfolio of lottery games is optimally priced. We estimate Barten’s synthetic differential demand system for the on-line lottery games operated by the Texas Lottery Commission. The demand system approach imposes theory-consistent demand restrictions that allow identification of parameters for games without price variation. We use the estimated parameters from the Barten model to construct expenditure and price elasticities. Results indicate that on-line games in Texas are generally substitutes for one another and the portfolio of games is not priced to maximize profit.

Keywords: gambling, lottery, consumer demand, elasticity, demand system estimation, Barten synthetic demand system

JEL Codes: D12, H21, H27, H71, L83

I. INTRODUCTION

Proceeds from lottery gambling are a significant source of revenue for U.S. state governments. As of 2012, all but seven states operate or sponsor lottery gambling with total sales of $68.7 billion and profits of $19.4 billion. In Texas, lottery gambling provides more revenue than excise taxes on alcohol or cigarettes, accounting for approximately 2 percent of total tax revenue (Texas Comptroller of Public Accounts, 2010). Although lottery games serve as a form of entertainment, they are established first and foremost as a source of revenue for the jurisdictions that operate them.

A number of studies estimate the elasticity of total sales with respect to effective price to investigate whether a lottery game maximizes expected profit (Cook and Clotfelter, 1993; Gulley and Scott, 1993; Scoggins, 1995; Farrell, Morgenroth, and Walker, 1999;
Farrell and Walker, 1999; Forrest, Gulley, and Simmons, 2000; Walker and Young, 2001; Dunn and Trousdale, 2014). But, most U.S. states operate multiple games simultaneously that differ with respect to jackpot size, odds of winning, and payout structure. Changes to the design of one game in a lottery portfolio, including the introduction of a new game,\(^2\) could influence the sales of the other games in the portfolio.

Although researchers have examined the extent to which lottery games are potential substitutes for one another (Forrest, Gulley, and Simmons, 2004; Grote and Matheson, 2006; Lin and Lai, 2006; Forrest and McHale, 2007; Guryan and Kearney, 2008; Humphreys and Perez, 2012),\(^3\) to our knowledge no one has estimated the substitution pattern for the entire portfolio of on-line games offered by a state-operated lottery in the United States. Therefore, we estimate consumer demand for the on-line lottery games offered by the Texas Lottery Commission (TLC) using the Barten synthetic demand system (Barten, 1993). Employing a demand system allows estimation of a full set of own-price, cross-price, and expenditure elasticities; this approach has been used to study price responsiveness for a wide range of goods including alcohol (Selvanathan, 1991; Clements and Salvanathan, 1991), meat (Nayga and Capps, 1994; Marsh, Schroeder and Minert, 2004), fish (Barten and Bettendorf, 1989), and carbonated beverages (Cotterill, 1994).

The benefits of adopting a formal demand system to study lottery pricing are twofold. First, economic theory provides parameter restrictions that identify the price-responsiveness of on-line games that do not exhibit price variation. This is an important contribution as on-line games with fixed nominal prices, fixed-odds, and fixed-payouts account for 40.2 percent of total on-line sales during our sample period. Second, it is possible to construct the profit maximization condition for the portfolio of on-line games offered by the TLC as a function of these elasticities. This allows us to test whether the TLC is pricing its on-line lottery games to maximize profit conditional on the design of its off-line gaming options, i.e., scratch-off lottery tickets.\(^4\)

\(^2\) The implications of adding new games to the lottery portfolio have been particularly salient to public finance in the aftermath of the 2008 financial crisis. In response to budget shortfalls, many states expanded their lottery options to increase revenue. For example, the multi-jurisdictional lottery games Powerball and Mega Millions had 31 and 12 participating states, respectively, in 2007. By 2010, this had jumped to 43 and 42 states, respectively.

\(^3\) These generally find that lottery games are either independent or slightly complementary to each other. Only Forrest, Gulley, and Simmons (2004) find substantial evidence of a substitution between games. Specifically, they report evidence that lotto games and instant-win games in the United Kingdom are partial substitutes. Their analysis also suggests that Wednesday and Saturday drawings of the same lotto game are substitutes for one another.

\(^4\) To our knowledge, none of the existing studies in the lottery demand literature investigate the objective function that lottery operators are attempting to maximize. To the extent that lottery gambling is viewed as a form of taxation, it is worth noting that taxes are not generally designed to maximize revenue. Yet, the existing literature focuses on net revenue maximization because lotteries are generally thought to operate as a business. Based on the TLC Mission Statement, this might not be an unreasonable standard: “We emphasize fiscal accountability by ensuring that all expenditures directly or indirectly generate revenue, enhance security, fulfill regulatory requirements, improve customer service and/or boost productivity. We recognize our responsibility in generating revenue for the State of Texas without unduly influencing players to participate in our games,” (“Our Core Values, Vision & Mission Statements,” TLC, http://www.txlottery.org/export/sites/lottery/About_Us/Core_Values/). Regardless of whether TLC is attempting to maximize profit, it is nevertheless an informative reference point and allows comparison with results from existing studies.
It is worth noting that while the subsequent analysis makes a valuable contribution to the existing literature on lottery pricing by examining the demand interactions in a portfolio of on-line games, there are a number of outstanding empirical issues that we are unable to address in this study. First, the designs of off-line games, e.g., scratch-off tickets and pull-tabs, are taken as given. Thus, while we can assess whether profit from on-line games is maximized, we cannot determine whether profit from all sources is maximized. Second, we do not have data to examine potential competition from illegal gambling activities, such as on-line sports betting.

To preview our main results, an analysis of aggregated time series data for sales of the six on-line Texas Lottery games in operation from 2006 to 2009 reveals that these games are generally gross substitutes for one another, though there is some evidence for complementarity among the three games with smaller top prizes. These results are drastically different from those found in previous studies highlighted above. We also find strong evidence that the current pricing strategy does not maximize profit from on-line games.

II. LOTTERY GAMBLING IN TEXAS

Texas is an ideal candidate for studying lottery demand for a number of reasons. First, Texas has relatively few legal gaming substitutes to the lottery, making it an isolated market where the sales of lottery tickets are unlikely to be influenced by other legal gambling activities. With the exception of six scattered racetracks and a lone Native American casino located in Eagle Pass at the border of Mexico, the lottery is virtually the only source of gambling available to the public within the state of Texas. Second, the vast majority of the population in the state is located in the interior, making the day-to-day cross-border shopping of lottery tickets a negligible issue. Third, the variety of available on-line lottery games offered by the Texas Lottery lends itself to the study of demand among games in portfolio.

Like most states, lottery gambling in Texas is a relatively new phenomenon. In November 1991, voters in Texas approved an amendment to the state constitution allowing state-operated lottery betting. A scratch-off ticket game went on sale in June 1992 and the first lotto drawing was held in November 1992. Initially operated by the Texas Comptroller of Public Accounts, in 1993 the state legislature ceded control of

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5 The demand for scratch-off lottery games has received relatively little attention in this literature. In large part, this gap arises because modeling scratch-offs is exceptionally difficult. States purvey a large number of games simultaneously with different ticket prices, prize designs, and odds of winning. Although this sounds similar to on-line gaming, there is a key difference. The winners of a particular lotto drawing are determined simultaneously. In contrast, the probability of winning a particular prize on a scratch-off ticket depends upon whether previous purchases have won that prize. Consider a simple game with one prize of $1 million. The expected value of playing that game increases monotonically as losing tickets are purchased until the winning ticket is revealed, at which point the expected value falls to zero. It is difficult to model player expectations when players have imperfect knowledge of whether prizes have already been claimed.

6 We acknowledge the fact that Texas’ neighboring states of Louisiana, Oklahoma, and New Mexico each have several gaming establishments that likely draw patronage of Texas residents. However, we assume that expenditure on the lottery is budgeted separately from that of out-of-state gambling and that market dynamics between the two are independent.
lottery gambling to the newly formed Texas Lottery Commission (TLC), who continues to operate all lottery gambling in the state.

The TLC purveys both on-line and off-line betting games. Off-line games, such as scratch-offs or pull-tabs, do not require the use of a computer terminal for purchase. Typically, a fixed number of tickets is printed for each game and the game lasts only as long as there are tickets to be sold. These games reveal instantly to the player whether they have won a prize.

On-line games require the use of a computer terminal to access a network through which the player’s bets are recorded. Although these games come in various forms (e.g. lotto, bingo, keno, and other numbers games), they generally involve having a player select a small group of numbers from a larger set with winners awarded prizes based upon how well their selection matches a randomly drawn result. In this paper, we restrict attention to the on-line games offered for sale in the state of Texas between April 2006 and December 2009. During this period, the TLC operated five on-line lottery games and participated in one multi-state lottery consortium, Mega Millions. Each game is described briefly below; full descriptions are provided in an Appendix available from the authors upon request.

A. Lotto Texas

Each ticket costs $1 and players select 6 numbers from a set of 54. The prize received for correctly selecting six, five, or four numbers follows pari-mutuel betting rules. That is, if multiple winning tickets are presented for a given prize tier, the prize pool allocated to that tier is split evenly among the winning tickets. The prize for correctly picking three numbers is guaranteed at $3. Drawings for Lotto Texas occur twice weekly, Wednesday and Saturday. If a drawing does not produce a jackpot winner, a portion of the sales are rolled over into the next drawing, causing the jackpot to increase.

B. Mega Millions

Each ticket costs $1 and players select six numbers: 5 from a set of 56 and 1 from a set of 46 (the Mega Ball). For an additional $1, players can also purchase a randomly selected multiplier that increases the value of non-jackpot prizes. Players who correctly pick each of the six numbers win a share of the announced jackpot. Lower tier prizes are guaranteed. Drawings for Mega Millions occur twice weekly, Tuesday and Friday. Jackpots are subject to rollover.

C. Texas Two Step

Each ticket costs $1. Players select five numbers: 4 numbers from a set of 35 and 1 bonus number, also from a set of 35. All prize tiers follow pari-mutuel rules except

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for winners who correctly pick one number from the first set and the bonus number or who pick no numbers from the first set and the bonus number, who receive guaranteed prizes of $7 and $5, respectively. Drawings for Texas Two Step occur twice weekly, Monday and Thursday. Jackpots are subject to rollover.

D. Cash 5

Each ticket costs $1. Players select 5 numbers from a set of 37. All prize tiers follow pari-mutuel rules except for winners who correctly pick two numbers, who receive a guaranteed prize of $2. Drawings for Cash 5 occur once daily except Sunday. Jackpots are subject to rollover.

E. Pick 3 and Daily 4

Players select either 3 or 4 numbers, each from a set of 10 (0 to 9). In addition to betting on which numbers are drawn, players can also wager on other characteristics, such as the order in which numbers are drawn (straight or box) or the sum of all the numbers drawn (Sum-It-Up). All prizes are guaranteed. The cost of play as well as the prize amount depend upon the game options chosen. Drawings take place twice daily except Sunday.

III. THE EFFECTIVE PRICE OF A LOTTERY TICKET

From the description of the lottery games above, it is evident that the nominal price of a lottery ticket — the amount of money a consumer pays to purchase a ticket — is typically constant. Yet, the good that lottery consumers are purchasing is not the physical ticket itself, but rather the opportunity to win a prize. The expected value of that opportunity will vary from drawing to drawing as other attributes of the game vary, e.g., the jackpot amount or the number of tickets purchased by other players. For example, as the jackpot gets larger, the amount of money a consumer can expect to win from purchasing a ticket will tend to increase. Conversely, as the number of tickets purchased increases, the probability of splitting the jackpot will also increase, thus decreasing the amount of money a consumer can expect to win. Thus, games that roll over unclaimed jackpots and utilize pari-mutuel payout schemes will experience different expected values from drawing to drawing. Economists therefore define an effective price that depends upon the expected value of each ticket purchased.

Cook and Clotfelter (1993) employ the expected average profit per ticket sold as their measure of effective price, \( P = P_n - EV \), where \( P_n \) is the nominal price of the ticket and \( EV \) is the expected value of playing. This definition has been widely adopted (Gulley and Scott, 1993; Scoggins, 1995; Farrell, Morgenroth, and Walker, 1999; Farrell and

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8 These rules are discussed in greater detail in the on-line Appendix A and are available in full at http://www.lottery.com/news/official_rules.cfm/GameID/TX4.
Walker, 1999; Forrest, Gulley, and Simmons, 2000) as it relates directly to the profit maximization problem of a lottery operator. Specifically, the coefficient estimate from a regression of log total sales on log average profit per ticket can be used to test whether the game is priced optimally.

Dunn and Trousdale (2014) point out several issues with this particular definition of effective price. Most importantly, drawings with large jackpots can exhibit expected values that exceed the nominal cost of a ticket. In such cases, the effective price is negative and thus its logarithm is undefined. These observations may be particularly important because they arise when the lottery controller sells the most tickets. Hence, they propose an alternative effective price definition, \( p = \frac{P_n}{EV} \), that is positive for all possible expected values and thus its logarithm is always defined. To ensure that the product of effective price and quantity equals total sales, they define effective quantity as \( q = Q \times EV \), i.e., \( EV \) chances to win $1.\footnote{Both Mason, Steagall and Fabritius (1997) and Garrett and Sobel (2004) use \( 1/EV \) in their demand model \( (P_n = $1) \) to preserve the inverse relationship between price and sales. In both cases, the choice of \( 1/EV \) appears to be based on goodness-of-fit. For example, Mason, Steagall and Fabritius (1997, p. 48) report, “Because this is a radical departure [from \( 1 – EV \)], we also analyzed some other polynomial specifications of price as a function of \( EV \). None of these alternatives, e.g., \( 1/EV^2, 1/(1 – EV) \), explained the variation in lotto sales nearly as well as did \( 1/EV \).”} Based on these definitions, Dunn and Trousdale (2014) demonstrate that how one handles drawings with super-unitary expected values observations can have a significant effect on elasticity estimates. In their study of Lotto Texas, they find that the estimated elasticity between effective price (defined as \( 1/EV \)) and effective quantity (defined as \( Q \times EV \)) was \(-1.40\) when these drawings were included in the sample, \(-1.30\) when the \( EV \) of these drawings were censored at unity, and \(-1.20\) when they were omitted. Therefore, in the subsequent analysis, the effective price definition of \( 1/EV \) is utilized to ensure that drawings with super-unitary expected values are included in the regression sample. We will also re-estimate the model omitting these drawings to examine whether dropping them from the regression sample would meaningfully influence coefficient estimates.

In addition to solving a practical estimation problem, this definition of effective price holds great intuitive appeal for the current application. Given a typical nominal price of $1, \( 1/EV \) defines the effective price as the cost of a chance to win $1. To illustrate, consider a drawing where the expected value is computed to be 50 cents. The player would then need to purchase two tickets, i.e., spend $2, to expect to win $1. This definition therefore standardizes the gambling options offered by the TLC — the price of every drawing for every game is defined as the cost of a chance to win $1. Of course, consumers need not be indifferent over games with identical expected values, as other attributes may influence their purchasing decisions, e.g., the riskiness of the bet or the mode of play.

The calculation of the effective price of a lottery ticket requires the calculation of the expected value of the associated drawing. We illustrate this calculation for a hypothetical drawing of Lotto Texas, where the advertised jackpot is $10 million and 2 million tickets are expected to be sold (calculations for the other games can also be found in Appendix A on the corresponding author’s webpage). Lotto Texas offers four different
prizes, which are awarded to players with tickets that correctly match 3, 4, 5, or 6 of the six officially drawn numbers which are selected at random from a field of 54. The estimated prize amount is positively related to the length of the odds. Matching all six numbers wins the jackpot at odds of 1:25,827,165. Matching 5 of 6 numbers wins the second-tier prize, which provides an average payout of $2,000 at odds of 1:89,678. Matching 4 of 6 numbers wins the third-tier prize, which provides an average payout of $50 at odds of 1:1,526. Finally, matching 3 of 6 numbers provides a fixed payout of $3 at odds of 1:75.

The top three prizes are pari-mutuel, which requires their respective prize pools to be split evenly among all winners. This leads to an interesting trade-off in the face of increased ticket sales; on one hand, as more people play, the amount a player stands to win will be larger, but on the other hand, it increases the chances a tie will occur (Cook and Clotfelter, 1993). This situation can be true even if jackpots are determined and advertised a period ahead, because the expected value is also a function of lower-tier prizes whose prize pools may be set as a fixed percentage of total sales. Operating under pari-mutuel rules allows a prize pool to be set as a fixed percentage of total sales, ensuring that the lottery will collect enough money to be able to pay out each prize, even in the event that a particular drawing experiences an uncharacteristically large number of winners. However, since the fourth-tier prize of $3 is guaranteed, the Texas Lottery operates a buffer account that can be used to pay all guaranteed obligations in the event the number of fourth-tier winners is abnormally large.

The total expected value of a bet is simply the sum of the expected value of the individual prize tiers. It is easiest to organize these prize tiers into three categories: jackpot, other prizes determined by pari-mutuel rules, and guaranteed prizes:

\[
EV = EV_{\text{jackpot}} + EV_{\text{other pari-mutuel}} + EV_{\text{guaranteed}}.
\]

Cook and Clotfelter (1993) point out that under the assumption that the numbers chosen by players are uniformly distributed across the set of all players, the expected share of a prize pool can be calculated using the Poisson approximation to the binomial distribution. Under the Poisson approximation, the expected number of winners in \(N\) independent random trials with probability \(\pi\) of success is \(N/(1 - e^{-\pi N})\). Thus, the expected value of an announced jackpot with \(N\) tickets purchased is

\[
EV_{\text{jackpot}} = \frac{CVo}{N} \left(1 - e^{-\pi N}\right).
\]

Since the jackpot typically advertised represents the value of a 20-year annuity, we use the present discounted value of this annuity (otherwise known as the cash value option,}

\[^{10}\text{Scoggins (1995) and Farrell, et al. (2000) find strong evidence for conscious selection, i.e., the distribution of numbers selected by lottery players is non-random. Yet, the latter report that conscious selection does not lead to significantly different estimation results. Baker and McHale (2009) find that while the higher moments of the distribution tend to be sensitive to the assumption of a Poisson distribution, the expected value of a ticket is typically over-estimated by 1 percent to 2 percent. In subsequent work, Baker and McHale (2011) provide a theoretical model to address the potential for conscious selection based on a modified mixture of the exponential and uniform distributions.}\]
or CVO). In our example, the expected value of a $10 million jackpot (CVO = $5.7 million) from a drawing with tickets sales of $2 million is 21 cents.

The pools for the second- and third-tier prizes are set at 2.23 percent and 3.28 percent of total sales, respectively. Thus, the expected value of these two prizes combined is:

\[
EV_{\text{other pari-mutuel}} = \frac{(0.0223)N}{N} \left(1 - e^{-\pi_eN}\right) + \frac{(0.0328)N}{N} \left(1 - e^{-\pi_eN}\right),
\]

or 6 cents. Finally, the probability of picking three numbers correctly is 1:75, so the expected value of guaranteed prizes is 4 cents (0.04=3/75). Therefore according to (1), the total expected value for the example drawing is 31 cents.

Based on this example, it is clear that there are only two possible sources of variation in the effective price given the payout and pricing structure determined by the TLC: the size of the jackpot, which varies according to whether previous jackpot prizes are rolled over, and the expected number of players, which determines both the expected probability of splitting the jackpot with other winners, as well as the expected size of lower-tier prize pools. To illustrate the relationship between jackpot size, expected value, and effective price, Figure 1 plots the latter two for all Lotto Texas drawings between May 9, 2009 and October 15, 2009. This period captures the longest stretch of drawings that failed to produce a winner. In addition to the drawing date, the x-axis is also labeled with the jackpot amount associated with that drawing. The drawing on May 6, 2009 produced a jackpot winner, causing the jackpot to reset to $4 million for the May 9 drawing. As subsequent drawings fail to produce a winner, the jackpot increased through rollovers, causing the expected value of a lottery bet to increase, and thus the effective price to decrease.

The jackpot for Lotto Texas eventually increased sufficiently so that the drawing on August 22, 2009 with a jackpot of $39 million exhibited an expected value greater than 1. Despite the fact that the cost of purchasing a ticket was less than the expected value of the purchase, the drawing did not produce a winner. Indeed, there would be 12 drawings with super-unitary expected values until a winner claimed a $76 million jackpot. Over the entire sample period, there were 32 Lotto Texas drawings and 12 drawings of Texas Two Step that exhibited a super-unitary expected value.

IV. PROFIT MAXIMIZATION BY THE LOTTERY OPERATOR

First, assume that the only cost associated with operating on-line gaming is paying the prize winnings (this assumption will be relaxed shortly). As demonstrated in Dunn and Trousdale (2014), the objective function for profit maximization can be expressed by

\[
\pi = P_n \times Q - Q \times EV = Q(P_n - EV) = q(p)(p - 1).
\]
Figure 1
Lotto Texas Jackpot Size and Expected Value for Drawings between May 9, 2009 and October 14, 2009

[Diagram showing the relationship between Expected Value and Effective Price (1/EV) over time.]
Maximizing (4) with respect to $p$ yields the following first order condition

$$\eta_{q,p} = -\frac{1}{1 - EV},$$

where $\eta_{q,p}$ is the effective price elasticity of demand. Thus the profit maximizing price results in an elasticity that varies with the expected value of the drawing. Extending this analysis to a portfolio of $N$ games, the objective function becomes

$$\max_{p_i} \pi = \sum_{i=1}^{N} \left[ p_i q_i(p_i) - q_i(p_i) \right] \quad \forall i \in N.$$ 

Maximizing (6) with respect to $p_i$ yields the first order condition

$$q_i + \sum_{j=1}^{N} p_j \frac{\partial q_j}{\partial p_i} - \sum_{j=1}^{N} \frac{\partial q_j}{\partial p_i} = 0.$$ 

This condition can be rewritten algebraically to become

$$\sum_{j=1}^{N} \left( \frac{w_j}{x} - \frac{q_j}{x} \right) \varepsilon_{ji} = -w_i,$$

where $x$ is total expenditure on all games, $w_j$ is the expenditure share for game $i$, and $\varepsilon_{ji}$ is the Marshallian price elasticity of demand for game $i$ with respect to game $j$. Re-indexing the above expression yields

$$\sum_{i=1}^{N} \left( \frac{w_i}{x} - \frac{q_i}{x} \right) \varepsilon_{ij} = -w_j \Rightarrow \sum_{j=1}^{N} \sum_{i=1}^{N} \left( \frac{w_i}{x} - \frac{q_i}{x} \right) \varepsilon_{ij} = -1,$$

which indicates that the optimal portfolio profit depends not only upon the individual price elasticities of each game, but also upon their related cross-price elasticities.

The expression in (9) was derived assuming that the lottery operators has no costs except for making payouts to winners. In practice, lottery operators do incur both administrative and operating expenses. For example, store owners who sell winning lottery tickets receive a percentage of the winnings from the TLC. Accounting for both the fixed and variable costs associated with operating a lottery would result in a profit maximizing elasticity of between $-1.1$ and $-1.2$ (Gulley and Scott, 1993).

V. DATA

The data for this study were collected from public records provided by the TLC and include daily ticket sales, advertised jackpots, projected ticket sales, and odds at each prize tier for each of six games spanning April 2006 to the end of 2009. TLC was only able to provide sales information for tickets purchased in Texas, thus total tickets sales for each Mega Millions drawing were calculated by summing total sales for each participating state.
In order to properly compare data among the six games, sales were aggregated to the level of the least frequently drawn games (i.e., twice weekly). Specifically, Monday to Wednesday was defined as one draw period and Thursday to Sunday was defined as another draw period. This allows each draw period to include every game at least once, and splits the number of drawings of the more frequent games (such as Cash Five, Pick 3, and Daily 4) in half. While no drawings are held on Sunday, tickets can still be purchased so any Sunday sales are lumped into the latter half of the week.

Descriptive statistics for the data are presented in Table 1. Lotto Texas is the largest in-state game operated by the Texas Lottery with tickets sales ranging from $1.4 million to $4.2 million per drawing and observed jackpots ranging from $4 million to $76 million over the sample period. It has the largest range of expected values among all the other games with a low of 17 cents and a high of $1.86, which translates to an effective price range of 54 cents to $5.81. We observe expected values greater than $1 for Lotto Texas, which occur when the jackpot is sufficiently large.

Ticket sales for Mega Millions are generally similar to Lotto Texas with a mean of $2.2 million per drawing. However Mega Millions experiences sharp spikes in sales for drawings with large jackpots creating a much larger range over the sample period: $1.1 million to $16.5 million. Multi-state participation allows Mega Millions to offer jackpots at a much larger scale than any single-state lotto game. As a result we observe jackpots ranging from $12 million to $370 million, with a mean of $66 million. Yet, the long odds of winning the jackpot make Mega Millions the least generous of the games, returning only 37 cents on average for every $1 bet.

Compared to Lotto Texas, Texas Two Step offers smaller jackpots, ranging from $0.2 million to $2.9 million, but at more favorable odds. This game only generates sales in the range of $0.3 million to $1.1 million, with a mean of $0.5 million (roughly one-fifth as large at Lotto Texas). However, Texas Two Step is the only game other than Lotto Texas that experiences expected values greater than $1 over the sample period.

Among the games using pari-mutuel betting rules, Cash 5 exhibits both the lowest jackpots and ticket sales. Jackpots range from $17,000 to $36,000, but have much more favorable odds at 1:435,897. Cash 5 is unique among the other games in the portfolio in that it does not use rollovers; if the top prize is not won, its prize pool is distributed among the second-tier winners. Since the top prize of Cash Five does not roll over, its effective price variation comes directly from the variation in sales from drawing to drawing, leading to relatively little variation in effective price ($1.80 to $2.12). Cash 5 has the highest overall expected value, paying out 51 cents for every $1 bet, on average.

Despite its relatively low payout, Pick 3 is the most popular on-line game in the portfolio generating average sales of just under $3 million tickets per drawing period, which is mostly likely due to its more favorable odds. Since Daily 4 is essentially a scaled version of Pick 3, both have a fixed effective price of $2. Sales of Daily 4 are roughly one-quarter those of Pick 3, likely reflecting that Daily 4 attracts a niche group of players with a slightly higher degree of risk tolerance than the average Pick 3 player.
Table 1
Descriptive Statistics

Post Lotto Texas Rule Change (N=385)

<table>
<thead>
<tr>
<th>Ticket Sales ($)</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lotto Texas</td>
<td>2,107,000</td>
<td>491,836</td>
<td>1,377,000</td>
<td>4,200,000</td>
</tr>
<tr>
<td>Mega Millions</td>
<td>2,206,000</td>
<td>1,467,058</td>
<td>1,134,000</td>
<td>16,500,000</td>
</tr>
<tr>
<td>Texas Two Step</td>
<td>473,000</td>
<td>150,316</td>
<td>293,000</td>
<td>1,159,000</td>
</tr>
<tr>
<td>Cash Five</td>
<td>250,000</td>
<td>42,895</td>
<td>135,000</td>
<td>380,000</td>
</tr>
<tr>
<td>Pick 3</td>
<td>2,862,000</td>
<td>213,066</td>
<td>1,888,000</td>
<td>3,491,000</td>
</tr>
<tr>
<td>Daily 4</td>
<td>528,000</td>
<td>94,656</td>
<td>332,000</td>
<td>1,158,000</td>
</tr>
</tbody>
</table>

| Top Prize ($)    |           |                    |         |         |
| Lotto Texas      | 17,200,000| 14,533,925         | 4,000,000 | 76,000,000 |
| Mega Millions    | 62,100,000| 57,809,074         | 12,000,000 | 370,000,000 |
| Texas Two Step   | 423,000   | 373,679            | 200,000  | 2,900,000  |
| Cash Five        | 27,500    | 3,286              | 17,000   | 36,000    |
| Pick 3           | 500       | 0                  | 500      | 500       |
| Daily 4          | 5,000     | 0                  | 5,000    | 5,000     |

| Expected Value ($) |       |       |       |       |
| Lotto Texas       | 0.48   | 0.33  | 0.17  | 1.86  |
| Mega Millions     | 0.37   | 0.15  | 0.22  | 0.96  |
| Texas Two Step    | 0.46   | 0.16  | 0.36  | 1.50  |
| Cash Five         | 0.51   | 0.03  | 0.47  | 0.56  |
| Pick 3            | 0.50   | 0.00  | 0.50  | 0.50  |
| Daily 4           | 0.50   | 0.00  | 0.50  | 0.50  |

| Effective Price ($) |       |       |       |
| Lotto Texas        | 2.81  | 1.35  | 0.54  | 5.81  |
| Mega Millions      | 3.13  | 1.03  | 1.05  | 4.59  |
| Texas Two Step     | 2.34  | 0.51  | 0.69  | 2.81  |
| Cash Five          | 1.98  | 0.10  | 1.80  | 2.12  |
| Pick 3             | 2.00  | 0.00  | 2.00  | 2.00  |
| Daily 4            | 2.00  | 0.00  | 2.00  | 2.00  |

<table>
<thead>
<tr>
<th>Odds</th>
<th>Draw Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lotto Texas</td>
<td>1:25,827,165</td>
</tr>
<tr>
<td>Mega Millions</td>
<td>1:175,711,536</td>
</tr>
<tr>
<td>Texas Two Step</td>
<td>1:1,832,600</td>
</tr>
<tr>
<td>Cash Five</td>
<td>1:435,897</td>
</tr>
<tr>
<td>Pick 3</td>
<td>1:1,000</td>
</tr>
<tr>
<td>Daily 4</td>
<td>1:5,000</td>
</tr>
</tbody>
</table>
We therefore combine these into a composite game, which is reasonable since Daily 4 is simply a scaled version of Pick 3.11

In Figure 2, we plot the total sales of each of the six games by drawing. This plot clearly illustrates the relative magnitudes and variability of sales among the games. Lotto Texas tends to perform better than Mega Millions in most periods, except when the Mega Millions jackpot is especially large. In these drawings the sales of Mega Millions tickets rise very sharply as depicted by the tall peaks. Sales for Cash 5 and Daily 4 are relatively flat.

To summarize, it appears that players tend to favor games at the tail ends of the risk spectrum. A small chance of winning big appears to be just as attractive in aggregate to a relatively good chance at winning a small amount with only limited taste for games that fall in the middle.

VI. ECONOMETRIC MODEL

In this study, own-price, cross-price, and expenditure elasticities are estimated using Barten’s synthetic differential demand system (Barten, 1993). In the consumer demand analysis framework, the demand system is typically viewed as an approximation to an unobserved demand relationship. The more flexible the specification, the better the approximation will be. But, greater flexibility comes at the cost of more parameters to estimate. A major advantage of the Barten synthetic system is that it parsimoniously generalizes four of the most commonly used specifications in demand analysis: the Rotterdam model (Barten, 1969; Theil, 1965), the first-differenced Linear Approximate Almost Ideal Demand System or FDLA-AIDS (Deaton and Muellbauer, 1980), the (Dutch) Central Bureau of Statistics or CBS model (Keller and van Driel, 1985), and the Neves NBR model (Neves, 1987). Thus, the Barten synthetic system should provide a better approximation to the true elasticities than relying on one of the underlying demand systems that it subsumes. In addition, the model is both linear in its parameters, so that it is relatively easy to estimate, and allows for the first-differencing of variables to address issues of non-stationarity (Matsuda, 2005).

To derive Barten’s synthetic demand system with respect to the effective prices of the five games offered by the TLC, we follow the notation in Matsuda (2005). Let \( p = (p_1, \ldots, p_5) \) be a vector of effective prices and \( m \) be total expenditure on all five games. Totally differentiating the effective Marshallian demand \( q_i(p,m) \) yields

\[
dq_i(p,m) = \frac{\partial q_i(p,m)}{\partial m} dm + \sum_{j=1}^{5} \frac{\partial q_i(p,m)}{\partial p_j} dp_j, \quad i = 1, \ldots, 5.
\]

The TLC began offering Daily 4 in September 2007. Although the entry of Daily 4 does not change the expected value of the composite good, it does influence other moments of the expected payout distribution. We examined robustness to this particular issue by re-estimating our model separately for the before and after sample periods. Our results did not change.
Figure 2
Lottery Sales ($Million) by Date
The Slutsky equation relates expenditure-constant Marshallian demand with utility-constant Hicksian demand

$$\frac{\partial q_i(p,m)}{\partial p_j} = \frac{\partial h_i(p,u)}{\partial p_j} - \frac{\partial q_i(p,m)}{\partial m} q_j(p,m),$$

where \( h_i(p,u) \) represents the Hicksian demand of game \( i \). Totally differentiating the budget constraint (adding-up condition), \( m = \sum_{i=1}^{5} p_i q_i \), yields

$$\sum_{i=1}^{5} p_i dq_i = dm - \sum_{i=1}^{5} q_i dp_i.$$

Substituting (11) and (12) into (10) and multiplying both sides by \( p_i/m \) yields

$$w_i d \log q_i = p_i \frac{\partial q_i}{\partial m} d \log \hat{Q} + \sum_{j=1}^{5} p_j \frac{\partial h_i}{\partial p_j} d \log p_j.$$

On the right-hand side of this equation, \( w_i d \log q_i \) represents the log-change in effective quantity of game \( i \), weighted by its expenditure share, \( w_i \equiv (p_i q_i)/m \). On the left-hand side, \( p_i (\partial q_i/\partial m) \) denotes the marginal expenditure share of game \( i \) and determines how additional expenditure to this game is allocated; \( d \log \hat{Q} \) denotes the Divisia volume index number for changes in real income. Note the use of the tilde on \( \hat{Q} \) is intended to differentiate this term from that used to denote the total tickets sales quantity, \( Q \), and \( (p_j p_i/m)(\partial h_i/\partial p_j) \) denotes the \( ij \)th element of the Slutsky matrix and pertains to the substitution effect between games \( i \) and \( j \) in response to a change in game \( j \)'s effective price (represented by \( d \log p_j \)).

Define the following:

\[
\alpha_i = (1 - \lambda) b_i + \lambda c_i \quad \text{(a vector of expenditure coefficients)},
\]

\[
\beta_{ij} = (1 - \mu) s_{ij} + \mu r_{ij} \quad \text{(a matrix of price coefficients)}, \quad \text{and}
\]

\[
r_{ij} = s_{ij} + w_i (\delta_{ij} - w_j), \quad \text{where} \quad \delta_{ij} \text{ is the Kronecker delta}.
\]

Assuming that \( p_i (\partial q_i/\partial m) = (\alpha_i + \lambda w_i) \) and \( (p_j p_i/m)(\partial h_i/\partial p_j) = \beta_{ij} - \mu w_i (\delta_{ij} - w_j) \), we obtain the final estimating equation for good \( i \):

$$w_i d \log q_i = (\alpha_i + \lambda w_i) d \log \hat{Q} + \sum_{j=1}^{5} [\beta_{ij} - \mu w_i (\delta_{ij} - w_j)] d \log p_j.$$

The estimated values for the parameters \( \lambda \) and \( \mu \) can be used to test whether one of the four demand systems is appropriate. If \( \lambda = 0 \) and \( \mu = 0 \), the general model reduces to the Rotterdam model. If \( \lambda = 1 \) and \( \mu = 0 \), the general model reduces to the CBS model. If \( \lambda = 1 \) and \( \mu = 0 \), the general model reduces to the NBR model. If \( \lambda = 1 \) and \( \mu = 1 \), the general model reduces to the differential LA-AIDS model. It is important to note that estimation using Barten’s synthesis does not impose any restrictions on the parameters \( \lambda \) and \( \mu \). Consequently, it is not necessary for the general model to fully reduce at all and can therefore be utilized as a demand system in and of itself (Brown, Lee, and Seale, 1994).
In addition to the parameters λ and μ, which enter the estimating equation for each good in the system, every equation includes one α and five β’s. In a five-good system, that results in 32 unknown parameters.

As a complete demand system, an accounting identity implies that the sum of sales for each lottery game must equal total lottery sales. This accounting identity implies the following adding-up restrictions: \( \sum \beta_g = 0 \) and \( \sum \alpha_i = 1 - \lambda \). For practical purposes, this adding-up condition implies that one equation is dropped from the system during estimation, as all of the parameters from that equation are linear combinations of parameters from the remaining equations. The choice of equation will not affect coefficient estimates; we omit the demand equation for the composite game, i.e., Pick 3 and Daily 4.

Estimating each of the remaining parameters without additional restrictions requires that the effective price of each on-line lottery game must vary during the sample period. While jackpot roll-overs and pari-mutuel payout rules generate variation in the effective price of Mega Millions, Lotto Texas, Texas Two Step, and Cash 5, there is no variation in the effective price of the composite good because its component games, Pick 3 and Daily 4, are fixed-odds, fixed-payout. In practice, this implies that \( \{\beta_{\text{composite}}\} \) cannot be recovered without further restrictions (recall that \( \{\alpha_{\text{composite}}\} \) and \( \{\beta_{i,\text{composite}}\} \) for \( i \neq \text{composite} \) are recovered from the adding-up condition provided by the accounting identity). Yet, economic theory provides additional parameter restrictions that can be used to recover the full set of parameters.

Assuming that consumers possess a rational preference ordering, the demand relationship must exhibit both homogeneity and symmetry. Algebraically, homogeneity and symmetry impose the following restrictions on the Barten synthetic demand system

\[
\sum_{j} \beta_{g} = 0 \quad \text{(homogeneity)}
\]
\[
\beta_{g} = \beta_{ji} \quad \text{(symmetry)}.
\]

The homogeneity restriction addresses two issues. First, imposing homogeneity overcomes the potential endogeneity of total lottery expenditure (Attfeld, 1985). Second, within each equation, the price effect of the \( n \)th game is a linear combination of the price effects of the other \((n - 1)\) games. Thus, one can identify the price effect of one game that does not exhibit price variation so long as the other games do exhibit price variation. Notice that given the accounting identity for adding-up/symmetry implies homogeneity along with additional parameter restrictions. Since symmetry is a stronger restriction than is necessary to recover the full set of unknown parameters, the subsequent analysis only imposes homogeneity.

Intuitively, homogeneity is the property that demand remains the same when prices and expenditure both double. Geometrically, an equiproportionate change in both prices and expenditure does not change the budget constraint. This property is often termed, “the absence of money illusion.” In typical demand analyses, where the prices of all goods vary, it is possible to test the validity of the homogeneity restriction. In the current study, however, because there is no price variation in the composite game, one cannot estimate the unrestricted version of the model.
Demand for Lottery Gambling

Despite the inability to test for homogeneity, its imposition is relatively costless in this setting. First, because there is no variation in the effective price of the composite good, imposing homogeneity as a restriction during estimation does not affect the estimated parameters on effective prices that do exhibit variation. Thus, we can still confidently examine the implications of price changes in the other games.

Second, there is no better alternative to recovering parameters on the price of the composite good until the TLC changes the payout or ticket price of Pick 3 and Daily 4. Hence, the choice is not between having an estimate of the own-price elasticity of the composite game with homogeneity and an estimate without homogeneity, but rather between having the former or no elasticity estimate at all. As the homogeneity restriction tends not to have substantial impacts on coefficient estimates even when it is rejected (Barten, 1969), using it to provide a first estimate of the price elasticities for the composite good seems no more or less unreasonable than other instances where economists deploy model restrictions to recover parameters that are otherwise observable, e.g., assuming a Cobb-Douglas production function to estimate total factor productivity.

A. Elasticity Calculations

Using the estimated parameters in the Barten model, it is possible to construct the full set of expenditure and price-elasticities. Unlike the log-log specification typically used in this literature, the Barten model generates elasticities that are non-constant. For example, the expenditure elasticity of demand with respect to game $i$ is given by

$$e_i = \frac{\alpha_i + \lambda w_i}{w_i},$$

and measures the responsiveness of demand to changes in expenditure on lottery games. Although the estimated parameters $\alpha_i$ and $\lambda$ are constants, the expenditure elasticity is a non-linear function of the budget share. It is thus possible to calculate the elasticity at any value of the budget share. In the results reported subsequently, this is done at the mean budget share.

As with all demand systems, the Barten model estimates expenditure-constant demand. That is, price changes are assumed to reallocate consumption across goods in the system, but do not affect total expenditure on goods in the system (we subsequently demonstrate how to construct total elasticities from the expenditure-constant elasticities). The compensated and uncompensated expenditure-constant elasticities are given by

$$\eta_{ij} = \frac{\beta_{ij}}{w_i} - \mu (\delta_{ij} - w_j),$$

$$\epsilon_{ij} = -\left(\frac{\alpha_i + \lambda w_i}{w_i}\right) w_j + \frac{\beta_{ij}}{w_i} - \mu \left(\delta_{ij} - w_j\right),$$

respectively, where $\delta_{ij}$ again denotes the Kronecker delta.
It is worth noting which elasticity calculations require the additional restriction of homogeneity and which are recoverable solely with the available price and expenditure variation, along with the adding-up restrictions. Variation in expenditure is sufficient to recover each of the $a_i$ and thus sufficient to recover each $e_i$. Variation in effective price is sufficient to recover each $\beta_{ij}$ for $i, j \neq \text{composite}$ and thus sufficient to recover each $e_{ij}$ for $i, j \neq \text{composite}$. The adding-up restriction provides $\{\beta_{i, \text{composite}}\}$ for $i \neq \text{composite}$ and thus $\{e_{i, \text{composite}}\}$ for $i \neq \text{composite}$. Therefore, without imposing homogeneity, we are nevertheless able to recover the expenditure elasticity of the composite game and its demand elasticity with respect to the prices of other games. The additional restriction of homogeneity provides $\{\beta_{\text{composite, } j}\}$, and thus makes it possible to recover the own-price elasticity of demand for the composite game, along with demand elasticities of the other games with respect to the effective price of the composite game.

To calculate the total uncompensated price elasticities we must also consider how price changes influence expenditure on lottery gambling (Edgerton, 1997). Defining $\xi_{ij}$ as the total uncompensated price elasticity, Edgerton (1997) demonstrates that

$$\xi_{ij} = \frac{d \log X}{d \log p_j} e_i = \frac{d \log X}{d \log P} \frac{d \log P}{d \log p_j} e_i = \epsilon_{ij} + \phi \pi_j e_i,$$

where $e_i$ and $\epsilon_{ij}$ are the expenditure elasticity (15) and expenditure-constant price elasticities (17), respectively, $\log P$ is the aggregate price index for on-line lottery gambling, and $\phi$ is the elasticity of total expenditure with respect to the price index; $\phi$ is easily recovered from an ordinary least squares (OLS) regression of log total expenditure on the log of the aggregate price index for lottery gambling. As shown in the on-line Appendix B, using the Stone price index as the aggregate price index for lottery gambling, $\log P = \Sigma w_i \log p_i$, yields the following expression for $\pi_j$

$$\pi_j = \frac{d \log P}{d \log p_j} = \frac{\omega_j + \sum_k \epsilon_{ik} \omega_k \log p_k}{1 + \sum_k \phi (1-e_k) \omega_k \log p_k}.$$

The numerator in (19) captures price effects on the price index. The first term is the direct effect of an increase in price ($\log p_j$) on the price index ($\log P$). The second term is the indirect effect of an increase in $\log p_j$, which causes substitution across other games, thereby altering the budget shares in the price index. The denominator in (19) captures expenditure effects on the price index, i.e., it accounts for the effects of changes in total expenditure on the budget shares in the price index. To see this intuitively, consider two scenarios:

1. If $\phi = 0$, then changes in log $P$ do not influence total expenditure, the denominator becomes unity, and the price index only changes because of price effects, and
2. If $\epsilon_{ik} = 1$, for all $k$, then expenditure expansion paths are linear, so changes in total expenditure do not influence budget shares, the denominator becomes unity, and the price index only changes because of price effects.
B. Endogeneity of Effective Price

The expected value of a lottery game that employs pari-mutuel payout rules depends upon expectations of total sales. If the expected value, and thus the effective price, is calculated using realized sales, then regressing effective quantity on the effective price will suffer from endogeneity as actual sales enter both the right-hand and left-hand sides of the equation. To address this issue, we follow Dunn and Trousdale (2014) by calculating the effective price of games operated by the TLC from the ex-ante sales predictions that are posted on their website. For Mega Millions, which is multi-jurisdictional, actual sales are used since sales from Texas account for a small percentage of total sales.

VII. RESULTS

Table 2 reports parameter estimates from the Barten synthetic demand system. These are difficult to interpret on their own, but it is worth noting that the estimates of λ and µ would lead us to reject the hypothesis that any of the nested demand systems are appropriate (see footnote 11). The top panel of Table 3 reports expenditure elasticities calculated according to (15) using the parameter estimates from the Barten synthetic demand system and the mean budget share. Each entry is interpreted as the percentage change in expenditure on the game listed in the column heading if total expenditure on lottery gambling increased by 1 percent. It is evident that expenditure elasticities vary substantially across the on-line lottery games offered by the TLC. Interesting, all but Mega Millions exhibit sub-unitary expenditure elasticities, indicating that sales on these games respond less than proportionally to changes in total expenditure on lottery gambling. In contrast, Mega Millions has an expenditure elasticity above one, indicating a higher degree of expenditure sensitivity. Each of these estimates is statistically significant at the 1 percent level.

In Table 3, we report the uncompensated price elasticity matrix calculated according to (19). In addition to the parameter estimates from the Barten model and the mean budget shares, these calculations also require knowledge of φ, the elasticity of total expenditure on lottery gambling with respect to the aggregate price index. Regressing the log of total expenditure on the Stone price index results in an estimated value of –0.635 with a standard error of 0.024. Own-price elasticities are marked in bold along

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13 Dunn and Trousdale (2014) demonstrate that the endogeneity of effective price is an issue in theory rather than in practice. They estimate the magnitude of endogeneity bias by comparing results under three specifications: effective price is calculated using actual sales, effective price is calculated using predicted sales from the lottery operator, and effective price is instrumented using the method suggested by Gulley and Scott (1993). They find that the estimated elasticities are very similar regardless of method used.

14 Notice that a regression of log effective quantity, \( q = EV \times Q \), on log effective price, \( p = 1/EV \), does not present additional endogeneity issues because \( EV \) enters both. It is simple to show that if \( \log q_i = \beta_q \log q_j + \beta_p \log p_j \) and \( \log Q_j = \gamma_q \log EV + \gamma_p EV \), then \( \beta_q = -\gamma_q \) and \( \beta_p = -(\gamma_q + 1) \). The mechanical relationship between \( q \) and \( p \) only influences the interpretation of the parameter estimates.

15 For each drawing period, we calculate the percentage of total expenditure attributable to each game, i.e., the budget share. We then calculate the mean budget share for each game over all drawing periods.
the diagonal of the matrix. Each game exhibits elastic demand with the exception of Cash 5, which yields a highly inelastic own-price estimate of –0.052. Each own-price estimate is statistically significant at the 1 percent level with the exception of Cash 5. The own-price elasticity estimate for the composite game (–1.009) is essentially unitary at the mean. Relative to the other games, Mega Millions and Texas Two Step exhibit the greatest responsiveness to changes in their own effective prices.

The uncompensated cross-price elasticity estimates give insight into the degree of substitutability between games in the face of income effects. These values are reported in the off-diagonal cells. Most of the cross-price terms are positive and statistically significant. Of the 18 elasticities that are statistically different from zero, 14 are positive, providing strong evidence of a generally substitutionary relationship among the games in the portfolio. Of particular note, the price changes for Mega Millions appears to have the largest relative effect on the quantity demanded of other games, with cross-price elasticity estimates ranging from 0.200 to 0.255. This may be driven, in part, by the game’s wide range of advertised jackpots over the period.
It is worth briefly discussing the estimation results when the drawings with super-unitary expected values are omitted from the analysis (available from the authors upon request). Failure to include such drawings has a significant influence on the estimated expenditure elasticities for both Lotto Texas and Mega Millions. The expenditure elasticity for the former increases to 1.80 from 1.47 when these drawings are omitted, while the expenditure elasticity for the latter falls from 3.43 to 2.98. Intuitively, drawings with large expected values will tend to generate the most ticket sales for that game. If that game also tends to account for a large share of total revenue, total sales will also tend to be highest. As Lotto Texas experienced 32 drawings when the expected value of playing was greater than the nominal cost of the ticket, omitting these drawings removed from the regression sample observations when both sales of Lotto Texas and total expenditure were atypically high, causing the expenditure elasticity for Lotto Texas to be under-estimated. Further, because Lotto Texas and Mega Millions tend to be substitutes for each other, omission eliminated observations when sales of Mega Millions were ceteris paribus lower, causing the expenditure elasticity of Mega Millions

<table>
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<tr>
<th>Expended Elasticities</th>
<th>Lotto Texas</th>
<th>Mega Millions</th>
<th>Two Step</th>
<th>Cash 5</th>
<th>Composite</th>
</tr>
</thead>
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<td></td>
<td>0.718*</td>
<td>1.675*</td>
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<td>(0.105)</td>
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<table>
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<th>Uncompensated Price Elasticities</th>
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<th>Two Step</th>
<th>Cash 5</th>
<th>Composite</th>
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</thead>
<tbody>
<tr>
<td>Lotto Texas</td>
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<td>0.033**</td>
<td>–0.558*</td>
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<td>Mega Millions</td>
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<td>–1.260*</td>
<td>0.093*</td>
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<td>–0.738*</td>
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<tr>
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<td>(0.013)</td>
<td>(0.041)</td>
<td>(0.174)</td>
</tr>
<tr>
<td>Two Step</td>
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<td>0.851*</td>
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<tr>
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<td>(0.031)</td>
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<td>(0.110)</td>
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<tr>
<td>Cash 5</td>
<td>–0.046*</td>
<td>0.170*</td>
<td>0.008</td>
<td>–0.052**</td>
<td>–0.870*</td>
</tr>
<tr>
<td></td>
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<td>(0.017)</td>
<td>(0.011)</td>
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<td>(0.078)</td>
</tr>
<tr>
<td>Composite</td>
<td>0.460*</td>
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<td>–0.011</td>
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<td>–1.009*</td>
</tr>
<tr>
<td></td>
<td>(0.065)</td>
<td>(0.079)</td>
<td>(0.012)</td>
<td>(0.012)</td>
<td>(0.060)</td>
</tr>
</tbody>
</table>

Notes: N = 385. The composite lottery game combines Pick 3 and Daily 4, the latter being a scaled version of the former. Standard errors are in parentheses. Asterisks denote significance at the 1 percent (*) and 5 percent (**) levels. Own-price elasticities are shown in bold.
to be over-estimated. These results provide further evidence that researchers must be particularly careful of omitting drawings associated with large jackpots.

Using these elasticity estimates from Table 3 to test for whether the portfolio of games offered by the TLC is priced to maximize profit according to (9), we find that the weighted sum of price-elasticities is −0.54, which is significantly different from profit-maximizing value of −1.0 at the 1 percent level. This result indicates that the Texas Lottery Commission is not optimizing the pricing structure of their games.

VIII. CONCLUSION

In this paper, we adopt a demand system approach to estimate the expenditure, own-price, and cross-price elasticities of demand for lottery games in Texas. This empirical framework allows us to incorporate restrictions from economic theory on the estimated parameter estimates and thereby estimate the price responsiveness for a composite game that did not exhibit any price variation but accounts for a substantial portion of total lottery sales. This is a methodological approach that could be adopted by other researchers to improve our understanding of how the on-line lottery games within an operator’s portfolio of products interact with each other.

We find that there is evidence of a mostly substitutionary relationship among the games offered by the TLC. This result is particularly interesting in comparison to findings from previous studies. The models developed by Forrest, Gulley, and Simmons (2004) and Perez and Forrest (2011) report small and statistically insignificant cross-price estimates, suggesting that competing games tend to be largely independent of one another, while the findings of Grote and Matheson (2006) suggest a slight complementary relationship between closely competing games.

There are a number of possible explanations for these differing conclusions. First, our empirical exercise does not omit game drawings when the expected value of playing surpasses the nominal price of purchasing a ticket. This is often necessitated by using the log of average expected profit per ticket as the definition of effective price (Cook and Clotfelter, 1993), but recent work (Dunn and Trousdale, 2014) has shown that estimation results are not robust to the omission of such drawings. Second, our analysis includes the full set of on-line games offered by the TLC, including a composite game constructed from two games that did not exhibit price variation. It is possible that failure to include all similar alternatives in a manner that permits full price interaction may have influenced previous estimation results.

In addition to purely econometric explanations, it possible that the TLC has simply designed a different portfolio of games than other lottery operators. Our results suggest that the pricing strategy employed by the TLC does not maximize total profit. Specifically, we estimate the elasticity of total sales with respect to the Stone price index to be −0.64 and calculate the weighted sum of the elasticity matrix to be −0.54, indicating that the portfolio of on-line lottery games is priced too low if the goal of the TLC is to maximize profit. Furthermore, we find a strong substitution pattern between Mega Millions and games operated by the TLC. Our results imply that a 10 percent decrease in the
effective price of Mega Millions is associated with a 1.7 percent decrease in the sales of Lottery Gambling

Lotto Texas and Cash 5 and a 2.3 percent decrease in sales of Texas Two Step. If a lottery operator maximizes profit by designing a portfolio in which increased sales of one game in response to a large jackpot does not cannibalize sales of other games that week, then cross-elasticities that are zero in Spain (Perez and Forrest, 2011) but positive in Texas suggests that the lottery in Spain may be better managed. Given the substitution pattern described above, however, it is not clear which games should be priced higher or by how much. Future work that develops new methods for determining how to optimally price a portfolio of games would therefore be an important advance in the literature.

Indeed, this issue highlights the principal result of our analysis: lottery games compete with each other and players are sensitive to relative price differences within a lottery market. This is a critical result for any lottery jurisdiction considering either altering the rules of existing games, adding new games to their portfolio, or joining a multi-state consortium. Because lottery ticket sales are a significant revenue source, imprecise estimates of demand can likely lead to sizable losses in potential revenue. Given that cross-price effects matter, demand for lottery games cannot be accurately modeled in isolation.

DISCLOSURES

The authors have no financial arrangements that might give rise to conflicts of interest with respect to the research reported in this paper.

REFERENCES


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