Multiple Modes of Tax Evasion: Theory and Evidence

Abstract - In this paper, we examine the theoretical and empirical implications of accounting for multiple modes of tax evasion. We find that increasing the probability of detection in a given mode has an ambiguous effect on compliance in the targeted mode as well as the untargeted mode. In order to gain greater insight into taxpayer behavior, we use the 1985 TCMP to estimate an empirical model with two modes of evasion. We find that increased enforcement effort has a positive effect on compliance in the targeted mode, a negative effect in the untargeted mode, and a positive overall effect on tax compliance.

INTRODUCTION

There is considerable evidence that taxpayers misreport some sources of income more intensively than others. For example, the Internal Revenue Service (IRS, 1996) estimates that 99.5 percent of wage and salary income is voluntarily reported by taxpayers for federal individual income tax purposes, while only 90.7 percent of realized long–term capital gains are voluntarily reported. The differential benefit–to–risk ratios of misreporting these line items may explain, at least in part, the observed patterns of voluntary reporting compliance by line item (see Joulfaian and Rider (1998)). More specifically, the higher statutory tax rate on wage income means that the tax benefit from misreporting this form of income in a given amount is greater than that from misreporting realized long–term capital gains in a similar amount. In contrast, underreporting realized long–term capital gains is less risky than underreporting wage income because the latter are subject to third–party reporting and withholding of tax, while the former generally are not. Since voluntary reporting compliance is higher for wages than long–term capital gains, the benefit–to–risk ratio appears to favor the underreporting of capital gains.

1 Also, non–compliance arising from negligence carries lighter penalties than non–compliance due to fraudulent behavior. U.S. taxpayers who understate their tax liabilities may be subject to civil or criminal penalties. Civil penalties are generally applied at a rate of 20 percent of the portion of the underpayment of tax resulting from a specified misconduct. In cases of fraud, which involve clear and convincing evidence that the taxpayer engaged in intentional wrongdoing, a criminal penalty may be applied. A willful attempt to evade tax is a felony.
Such differential patterns of tax and enforcement treatment also create opportunities for taxpayers to manage the risks of detection by misreporting a variety of line items. We refer to such strategies as multiple modes of tax evasion.

Generally speaking, theoretical and empirical studies of tax compliance assume a single mode of evasion. This literature provides valuable insights into taxpayer behavior and provides guidance for the proper conduct of tax administration. It should be recognized, however, that multiple modes of tax evasion greatly complicate the task of administering a tax system. For example, increasing the probability of detection in one mode may simply lead to decreased compliance in other modes. In fact, the resulting revenue increase from the mode targeted for increased enforcement effort may be more than fully offset by deteriorating compliance in others. Therefore, it is important to gain greater insight into the effect of tax enforcement on the optimal compliance behavior of taxpayers in more realistic settings with multiple modes of tax evasion.

We explore these issues by developing a theoretical model with two modes of tax evasion. We find that increased enforcement effort has an ambiguous effect on compliance in the targeted mode as well as the untargeted mode. In order to determine the effect of changing enforcement effort on tax compliance, we estimate simultaneous equations of income and deductions reporting compliance, using data from the Internal Revenue Service’s (IRS) 1985 Taxpayer’s Compliance Measurement Program (TCMP). Then, we use our estimates to simulate alternative enforcement strategies. We find that the net revenue effect of increased enforcement effort is positive: the revenue resulting from increased compliance in the mode targeted for increased enforcement effort is only partially offset by deteriorating compliance in the untargeted mode.

The remainder of the paper is organized as follows. In the next section, we briefly review the literature on tax compliance. In the third section, we present our theoretical model. We describe our empirical approach in the fourth section. In the fifth section, we discuss our empirical results. The final section summarizes our findings and offers suggestions for future research.

A BRIEF REVIEW OF THE LITERATURE

Given the number of papers on tax compliance, we cannot do justice here to the entire literature. Therefore, we limit this review to those articles that we regard to be the most relevant to the present study. In their seminal papers, Allingham and Sandmo (1972) and Yitzhaki (1974) assume a single mode of evasion with an associated detection probability and penalty. They find that these two tax enforcement parameters have a positive effect on compliance. In other words, an increase in enforcement effort unambiguously increases compliance. Pencavel (1979), Cowell (1985), and Sandmo (1981) add labor supply to the model, thus making income endogenous.


3 Cummings, Martinez–Vazquez, and McKee (2001) provide experimental evidence on compliance behavior with multiple modes of tax evasion. They find that increased enforcement effort in one mode of evasion can lead to lower overall tax compliance.

4 Andreoni, Erard, and Feinstein (1998) and Alm (1999) provide excellent reviews of the tax compliance literature.
In the case of endogenous income, the effect of the enforcement parameters on tax compliance is ambiguous.

Although several theoretical studies, including Pencavel (1979), Christiansen (1980), and Cowell (1981), distinguish between different forms of tax evasion, these studies do not explicitly recognize that taxpayers may employ them to manage the risks of detection. An important exception is Klepper and Nagin (1989) who examine line item reporting compliance. They argue that there is a substitution process for reporting compliance among line items. They acknowledge, however, that this result follows from the assumed functional form of the detection probabilities. In this paper, we use a more general framework, which allows us to consider the influence of attitudes toward risk and fixed and independent probabilities of detection on the optimal compliance behavior of taxpayers.

In addition to this study, two other papers consider multiple modes of tax evasion. Cremer and Gahvari (1994) derive optimal tax rules, and Cowell and Gordon (1995) derive optimal audit strategies. In contrast, we focus on the optimal reporting compliance of taxpayers. Clearly, all three perspectives are related to one another; however, each perspective is distinct and provides unique insights into the enforcement and policy implications of multiple modes of tax evasion.

The empirical literature also generally assumes a single mode of tax evasion. Using the 1969 TCMP and assuming a single mode of tax evasion, Clotfelter (1983) finds that income and detection probabilities have a positive and statistically significant effect on income reporting compliance, while the tax rate has a negative and statistically significant effect on compliance. Two papers, one by Klepper and Nagin (1989) and the other by Feinstein (1991), introduce models with multiple modes of tax evasion. Klepper and Nagin (1989) estimate separate equations for voluntary reporting percentages by line item using 1982 TCMP data aggregated by audit class. They include three classes of variables: measures of the cost to the IRS of establishing the true amount on each line item; measures of the complexity of the reporting rules pertaining to each line item; and measures of the ambiguity of the legal requirements for each line item. They find that these variables have a negative effect on line item reporting compliance. Although their data do not permit them to control for income or marginal tax rates, they report evidence in support of the hypothesized substitution process for line item reporting compliance described above.

In a path breaking study, Feinstein (1991) estimates a model of fractional detection using pooled 1982 and 1985 TCMP data. In contrast to Clotfelter (1983), Feinstein (1991) finds that the tax rate has a positive and statistically significant effect on compliance in the pooled regressions, but that the estimated effect of income is not statistically significantly different from zero. In order to measure the scope for evasion on a particular return, Feinstein (1991) includes dummy variables for the presence of sole proprietorship income (Schedule C) and farm income.

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5 Swary, Landskroner, and Paroush (1990) develop a portfolio approach to tax evasion where taxpayers make decisions on compliance including not only tax evasion but also risky and riskless financial assets. Thus, evasion is part of the portfolio decision of the taxpayer.

6 There also is a substantial game-theoretic literature on tax compliance, with different types of taxpayers. Graetz, Reinganum and Wilde (1986) are the first to incorporate honest taxpayers into a game-theoretic framework. Erard and Feinstein (1994) show the importance of the presence of honest taxpayers in a model with income distributed along a continuum rather than discretely, as in the case of Graetz et al. (1986). Reinganum and Wilde (1985) examine the case when the tax authority commits to an audit rule. Sanchez and Sobel (1993) provide an excellent discussion of this type of model.
(Schedule F). The idea is that it is more difficult for the IRS to detect misreporting of these sources of income because they are not subject to third-party reporting or withholding. As expected, these two variables have a negative and statistically significant effect on income reporting compliance.

Feinstein (1991) also estimates a disaggregated or multi-mode version of his partial detection model using the 1982 TCMP. This model consists of four equations: (1) an evasion equation for underreporting AGI; (2) an evasion equation for overstating deductions; (3) a detection equation for detecting underreported AGI; and (4) a detection equation for detecting overstated deductions. The stochastic disturbances are jointly normally distributed with correlation \( \rho_1 \) in the two evasion equations and jointly normally distributed with correlation \( \rho_2 \) in the two detection equations.\(^7\) In this version of the model, income and the marginal tax rate have a negative and statistically significant effect on reporting compliance in both equations.

The empirical literature reports mixed findings on the role of marginal tax rates and income on income tax reporting compliance.\(^8\) But there appears to be some consensus in this literature on the positive effect of lower costs of detection (diminished scope for evasion) on compliance. This conclusion is robust to alternative econometric specifications, variable definitions, and data.\(^9\) With the exception of Klepper and Nagin (1989) and Feinstein (1991), most of these studies do not allow us to draw conclusions about the effect of increased enforcement effort in settings with multiple modes of tax evasion. Our empirical approach focuses on such issues.

We improve upon the work of Klepper and Nagin (1989) and Feinstein (1991) in several ways. Klepper and Nagin use aggregate data and, thus, are unable to control for marginal tax rates and income. Omitting these regressors may lead to biased estimates if the audit probabilities are correlated with the omitted variables. Since we use individual tax return data, we are able to control for the potential effects of tax rates and income on reporting compliance. Although Feinstein (1991) makes a significant advance in the estimation of tax compliance models, he estimates the model on a relatively small number of returns (2,267) in order to make his model computationally tractable. In contrast, we estimate our model on 42,811 returns and more carefully control for the scope for tax evasion on a given return by taking advantage of third-party reporting of particular line items.

A MODEL OF MULTIPLE MODES OF TAX EVASION

Our basic research question concerns the effect of increased enforcement effort on tax evasion. In particular, we would like to know whether alternative modes of evasion are substitutes or complements and how this relationship affects the overall response of evasion to increased enforcement effort. We address these issues by introducing a second mode of evasion into the Allingham–Sandmo–Yitzhaki model of tax evasion.\(^10\)

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\(^7\) The correlation between the disturbances in the two evasion equations (\( \rho_1 \)) is estimated to be 0.8 and 0.2 in the case of the correlation between the detection equations (\( \rho_2 \)).

\(^8\) For example, Alm, Bahl, and Murray (1990) and Dubin and Wilde (1988) report a positive relationship between marginal tax rates and income reporting compliance; Feinstein (1991) reports mixed results; and Clotfelter (1983) and Joufi aian and Rider (1996, 1998) report a negative effect.

\(^9\) Experimental work also supports the conclusion that participants who have greater perceived opportunities for noncompliance tend to be significantly less compliant. See, for example, Robben et al. (1990).

\(^10\) In addition to viewing line items on a given return as different modes of evasion, an alternative interpretation, not pursued in this paper, is the decision to evade a variety of different taxes (e.g., corporate income tax, sales tax, payroll tax, and so on). Many of the questions we ask about multiple modes of evasion in the context...
We assume that individuals maximize expected utility ($EU$), which is a function of after–tax income ($Y - T$), where $Y$ is lump–sum income, and $T$ is the true tax liability. We also assume that the utility function is a monotone increasing and concave function of after–tax income, or $U' > 0$ and $U'' \leq 0$. In this framework, we assume that there are two modes that an individual can use to avoid reporting $T$: for example, understating true income and exaggerating true deductions. These two modes of underreporting tax liability are denoted $E_i$, where $i = 1, 2$. For the sake of simplicity, we assume an interior solution exists, or $0 < E_i < T$ and $0 < \Sigma E_i < T$ ($i = 1, 2$).\(^{11}\)

The probabilities of detection in modes 1 and 2 are denoted $P_1$ and $P_2$, respectively.\(^{12}\) We assume that the detection probabilities are fixed and independent, or $\partial P_i / \partial E_j = 0$, ($i, j = 1, 2$). In other words, the probabilities of detection are independent of both the level of evasion in the targeted mode as well as the level of evasion in the untargeted mode. Although this assumption does not describe the entire array of enforcement strategies employed by the IRS, it does capture essential features of the IRS’s information returns processing program (IRP). The IRP is by far the IRS’s most effective tax enforcement program in terms of tax assessments per enforcement dollar. Briefly, the IRP program consists of using high speed computers to match nearly one billion third–party reports of income and deductions to over one hundred million individual income tax returns.\(^{13}\) This match allows the IRS to identify anomalous reports and issue tax assessments based on those anomalies. For cost reasons, the IRS does not issue assessments for every anomalous report. Furthermore, the probabilities of detecting misstatements on line items covered by third–party reports are independent of one another and independent of detecting misreports on line items that are not covered by third–party reporting. In short, we believe that the assumption of fixed and independent detection probabilities captures essential features of the IRP program.\(^{14}\)

Following Yitzhaki (1974), we assume that taxpayers must pay a penalty that is proportional to the unreported tax liability. Specifically, the payment for detection in mode 1 is equal to the unreported tax liability ($E_1$) and a penalty $\theta_1 E_1$, where $\theta_1$ is the penalty rate for mode 1. Likewise, a taxpayer detected avoiding tax in mode 2 must pay $(1 + \theta_2)E_2$, where $\theta_2$ is the penalty rate for mode 2. Finally, we assume that $P_1 < P_2$ and $\theta_1 > \theta_2$ in order to ensure an interior solution ($0 < E_i < T$).

If both modes of evasion are used, there are four possible outcomes: (1) evasion is not detected in either mode, with

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\(^{11}\) The assumption that $0 < E_i < T$ for $i = (1, 2)$ is not required; however, it is a useful assumption for the purpose of interpreting the comparative–static results.

\(^{12}\) A detection probability is a compound event. Specifically, there is a probabilistic event that a return, or line item, will be selected for review. Conditional on being selected, some or all of the misreporting may be uncovered. For simplicity’s sake, we assume all evasion is uncovered if a return (line item) is selected for review.

\(^{13}\) In tax year 1985, the tax year from which our data are drawn, the IRS required third–party reporting of wage and salary income (IRS Form W2), interest and dividend income (IRS Form 1099), home mortgage interest income, and state and local taxes, the latter two being deductions on Schedule A of IRS Form 1040.

\(^{14}\) Obviously, this assumption does not completely describe the various enforcement strategies employed by the IRS, particularly special audits and field audits. Martinez–Vazquez (1995) examines alternative enforcement strategies, assuming multiple modes of tax evasion, such as independent and endogenous probabilities of detection and joint and endogenous probabilities of detection. The implications of these alternative models are very similar to those reached in the present paper.
probability \((1 - P_1)(1 - P_2)\); (2) evasion is detected in both modes, with probability \(P_1P_2\); (3) and (4) evasion is detected in one mode but not the other, with probabilities \((1 - P_1)P_2\) and \((1 - P_2)P_1\), respectively. Accordingly, expected utility \((EU)\) can be written as follows:

\[
[1] \quad EU = (1 - P_1)((1 - P_2)U(Z_1) + P_2U(Z_2)) \\
+ P_1((1 - P_2)U(Z_3) + P_2U(Z_4)),
\]

where \(Z_1 = Y - T + E\); \(Z_2 = Y - T + E\); \(Z_3 = Y - T - \theta_1E_1 + E_2\); and \(Z_4 = Y - T - \theta_2E_1 - \theta_2E_2\).

In the following discussion, we assume that an interior solution always exists. The first order conditions for an interior maximum of [1] are given as follows:

\[
[2] \quad \frac{\partial EU}{\partial E_1} = (1 - P_1)((1 - P_2)U'(Z_1)) \\
- P_2U'(Z_2) - \theta_1P_1((1 - P_2)U'(Z_3)) \\
+ P_2U'(Z_4) = 0;
\]

\[
[3] \quad \frac{\partial EU}{\partial E_2} = (1 - P_1)((1 - P_2)U'(Z_1)) \\
- P_2\theta_2U'(Z_2) + P_1((1 - P_2)U'(Z_3)) \\
- P_2\theta_2U'(Z_4) = 0.
\]

We assume that the second order conditions (see equations [A–4] through [A–6] of the Appendix) are satisfied by the concavity of the expected utility function.

The predictions of this model are formally described below; the details of important derivations are provided in the Appendix for interested readers. Generally speaking, we find in the theoretical model that income and the enforcement parameters have an ambiguous effect on compliance in both modes.

We begin by examining the effect of changes in income on tax compliance. By totally differentiating the first order conditions above with respect to \(Y, E_1\) and \(E_2\), we derive the following expression for the effect of income on the level of evasion in mode 1:

\[
[4] \quad \frac{dE_1}{dY} = -M^1P_2(1 - P_1)(1 - P_2)(1 + \theta_1U''(Z_1)) \\
- \theta_2U''(Z_2) - M^1P_1P_2(1 - P_2)(1 + \theta_2U''(Z_2)) \\
- P_1\theta_2(1 + \theta_2U''(Z_4)) > 0,
\]

where

\[
M = \left( \frac{\partial^2 EU}{\partial E_1^2} \right) \left( \frac{\partial^2 EU}{\partial E_2^2} \right) - \left( \frac{\partial^2 EU}{\partial E_1 \partial E_2} \right)^2 > 0.
\]

Scrutiny of expression [4] shows that an increase in income has an ambiguous effect on compliance in mode 1, even if we assume decreasing absolute risk aversion. We obtain a very similar expression for \(\frac{dE_2}{dY}\) and find that the effect of income compliance in mode 2 is also ambiguous. Although this result may be surprising, it closely parallels the result obtained by Allingham and Sandmo (1972) for a single mode of evasion. They find that the income effect on compliance is ambiguous when the penalty is less than 100 percent, even when they assume decreasing absolute risk aversion.

Turning to the effect of changing the probability of detection on compliance, we find that this effect is also ambiguous. In order to provide some intuition for this result, we derive the following expression:

\[
[5] \quad \frac{dE_1}{dP_1} = \left( \frac{D}{C} \right) \left( \frac{dE_1}{dY} \right) + \left( \frac{C}{A} \right) \left( \frac{dE_1}{dY} \right) \\
+ \left( \frac{D^2}{C} \right) \left( \frac{EU_{1i}}{M} \right) - \left( \frac{BC}{A} \right) \left( \frac{EU_{1i}}{M} \right),
\]

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where

\[ A = (1 - P_2)[(1 - P_2)U''(Z_1) + P_2U''(Z_2)] \\
- \theta_1P_1[(1 - P_2)U''(Z_3) + P_2U''(Z_4)] \geq 0; \]

\[ B = (1 - P_2)[(1 - P_2)U''(Z_1) - \theta_2P_2U''(Z_2)] \\
+ P_1[(1 - P_2)U''(Z_3) - \theta_2P_2U''(Z_4)] \geq 0; \]

\[ C = -(1 - P_2)[U'(Z_1) + \theta_1U'(Z_3)] - P_2[U'(Z_2) \\
+ \theta_1U'(Z_4)] < 0; \] and

\[ D = (1 - P_2)[U'(Z_1) - U'(Z_2)] + P_2\theta_1[U'(Z_4) \\
- U'(Z_4)] \leq 0. \]

Equation [5] shows that the effect of increasing the probability of detection on compliance in the targeted mode \( (dE_i/dY) \) can be expressed as a linear function of an income effect \( (dE_i/dP_i) \) and a substitution effect \( (dE_i/dP_1) \). As discussed in the Appendix, the sign of \( dE_i/dY \) is ambiguous; the signs of \( dE_i/dP_1 \), \( (D/C) \), \( (C/A) \) and the third term in square brackets on the right-hand-side of [5] are all ambiguous. Consequently, we cannot predict the sign of [5] based on the theory alone. We derive a similar expression to [5] for \( dE_2/dP_i \) and find that the effect of increasing the probability of detection in mode 1 on compliance in mode 2 is also ambiguous. Finally, an increase in the penalty of detection in a given mode has an ambiguous effect on compliance in both modes. This result is also described in the Appendix.

Given the theoretical ambiguity of the effect of changing enforcement effort on tax compliance, we conclude that these are ultimately empirical questions. We turn to a discussion of our main empirical findings in the following section.

A PRELIMINARY LOOK AT THE DATA

In our empirical analysis, we use a sample of returns constructed from the IRS’s 1985 TCMP data file, which is a stratified random sample consisting of approximately 49,162 federal individual tax returns that have been subject to thorough line by line review by experienced IRS tax examiners. The 1985 TCMP records the taxpayer’s report and the examiner’s correction for most line items on the return and accompanying schedules, providing extremely detailed information about reporting compliance for tax year 1985.\(^\text{15}\)

We make the following restrictions on the sample. In contrast to taxpayers facing positive marginal tax rates, taxpayers in the credit range of the earned income tax credit (EITC) face negative marginal rates and, therefore, have an incentive to overstate income in order to claim a larger credit.\(^\text{16}\) Therefore, we exclude taxpayers in the credit range of the EITC. Since married taxpayers filing jointly are subject to a different tax rate schedule than single taxpayers, we also eliminate returns in which the auditor-adjusted filing status differs from the reported one. For reasons related to the empirical specification, described in greater detail below, we also eliminate returns with negative adjusted gross income (AGI), as reported by the taxpayer.

\(^\text{15}\) There are three limitations of TCMP data. First, since the TCMP sample is drawn from tax filers, the data do not provide information on non-filers. Second, tax returns lack detailed information on demographic factors that may influence tax compliance behavior. Friedland, Maial, and Rutenberg (1978), Witte and Woodbury (1983), Baldry (1987), Dubin and Wilde (1988) and Beron, Tauchen, and Witte (1992) have examined the role of a variety of demographic variables on tax compliance, including age, race, and education among others. These studies generally have found that demographic variables are significant determinants of tax evasion. Third, it is well known that the TCMP fails to detect potentially significant amounts of underreported income, particularly income from sources that are not subject to third-party reporting. Despite these shortcomings, TCMP data are widely regarded to be the best source of data available on tax noncompliance.

\(^\text{16}\) See Joulfaian and Rider (1996) for an examination of the compliance effects of the EITC.
and corrected upon audit. The resulting sample consists of 42,811 returns.

As a first step, we examine our data for evidence of multiple modes of tax evasion by sorting our sample of returns into those that understate, correctly state, and overstate income and deductions. All nine possible strategies are employed in our sample; specifically, these include taxpayers who (1) correctly report income and deductions; (2) correctly report income and exaggerate deductions; (3) correctly report income and understate deductions; (4) understate income and deductions; (5) understate income and correctly report deductions; (6) understate income and exaggerate deductions; (7) exaggerate income and deductions; (8) exaggerate income and correctly report deductions; and (9) exaggerate income and understate deductions.

Strategies 2, 4, 5, 6, and 7 are consistent with understating taxable income; the others are not. We refer to strategies 2 and 5 as single mode strategies, and strategies 4, 6, and 7 as multi–mode strategies.

If we assume fixed and independent probabilities of detection, strategies 4 and 7 may not make much sense. Suppose, however, the probability of detection in a given mode depends upon the level of evasion in both modes. Under such circumstances, strategies 4 and 7 make more sense, if they reduce the probability of detection. For example, suppose the IRS audits 50 percent of the returns in which the ratio of itemized deductions to total income is 1.5 standard deviations greater than the mean, audits 75 percent of the returns when the ratio is two standard deviations greater than the mean, and so on. Now, suppose a taxpayer dramatically understates total income. In this case, exaggerating itemized deductions may significantly increase the probability of an audit. In fact, if the ratio of true contributions to reported income is sufficiently large, correctly reporting deductions may make an audit too likely for comfort. Under such circumstances, the taxpayer may rationally choose to reduce the risk of detection by understating deductions.

In the theory section, we do not consider more complex enforcement strategies like joint and endogenous probabilities of detection. We focus instead on fixed and independent probabilities of detection, which, as previously discussed, share important features with the IRS’s IRP program. In our empirical model, we use third-party reporting of particular line items to control for the scope for misreporting income and deductions on a return. Therefore, the assumption in our theoretical model of fixed and independent detection probabilities provides a good fit between the theory and our empirical approach.

Summary statistics for the full sample sorted according to compliance status are reported in Table 1A. For example, the cell in the third row section and third column of Table 1A shows that 41 percent of the returns with income understatements exaggerate deductions. For this subset of returns, the average understatement of income is $3,603, and the average overstatement of deductions is $1,647. Consequently, taxable income is understated by an average amount of $5,250 [\(-3,603 - 1,647\)], and average auditor-adjusted AGI for these returns is $65,402. The cell in the third row section and last column shows that 46 percent of all returns understate total income. Conditional on understating income, the

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17 Recall that the theoretical model was restricted for convenience of interpreting results by assuming taxpayers do not understate deductions or overstate income.

18 It is important to keep in mind, however, that the compliance patterns observed in the TCMP data reflect taxpayer responses to the complete array of enforcement strategies employed by the IRS. Although strategies 4 and 7 may not be equilibrium responses to fixed and independent probabilities of detection, they are noted here because they are observed in the data.
average understatement is $3,265; deductions are exaggerated by an average of $411; the average understatement of taxable income is $3,677; and the average auditor–adjusted AGI for this subsample is $54,777.

Table 1A provides ample evidence that taxpayers employ multiple modes of tax evasion. Approximately 24 percent (0.62*0.38) of returns correctly report taxable income, 63 percent overstate taxable income, and the remaining 13 percent overstated taxable income. Of the 63 percent that understate taxable income, approximately 57 percent—36 percent of all returns—use a multi–mode strategy. Two of the three multi–mode strategies are consistent with understating taxable income and, in these cases, the average understatement exceeds that for single mode strategies.

As previously discussed, there are lower costs to the IRS of detecting misstatements of tax attributes on line items covered by third–party reports. Therefore, we should expect greater compliance on those line itemized deductions + adjustments

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<th>Itemized Deductions + Adjustments</th>
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<th>Correctly state</th>
<th>Overstate</th>
<th>Row totals</th>
</tr>
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Correctly state

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Understate

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Column totals

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<th>Deductions gap</th>
<th>Taxable Income gap</th>
<th>AGI</th>
</tr>
</thead>
<tbody>
<tr>
<td>17%</td>
<td>–$3,121</td>
<td>–$1,318</td>
<td>–$2,670</td>
<td>–$2,120</td>
</tr>
<tr>
<td>–$2,813</td>
<td>0</td>
<td>$4,333</td>
<td>$1,086</td>
<td></td>
</tr>
<tr>
<td>–$308</td>
<td>–$1,318</td>
<td>–$7,003</td>
<td>–$3,206</td>
<td></td>
</tr>
<tr>
<td>$58,151</td>
<td>$33,587</td>
<td>$62,443</td>
<td>$48,298</td>
<td></td>
</tr>
</tbody>
</table>

^Income gap equals reported total income less corrected total income.
^Deductions gap equals reported deductions (+ adjustments) less corrected deductions (+ adjustments).
^Taxable income gap equals reported taxable income less corrected taxable income.
^AGI equals total income less adjustments.

We should not assume that all reporting errors are deliberate. After all, one would assume that the fact that some taxpayers have apparently overstated taxable income in many cases reflects honest mistakes. If all overstatements of taxable income are honest, then it seems reasonable to assume that accidental reporting errors are symmetric about zero and an equal number accidentally understate taxable income. It is clear from examination of Table 1A that more taxpayers understate taxable income than overstate it, and they do so in larger average amounts. Thus, it seems reasonable to conclude that at least some reporting errors are intentional. Nevertheless, some reporting errors, even those dramatically in the taxpayers favor, may reflect honest mistakes.
items that are subject to third–party reporting, like wage and salary income, than on those that are not. To gauge the impact that information reporting and withholding have on reporting compliance, we split the full sample into returns with auditor–adjusted non–wage income exceeding 25 percent of auditor–adjusted total income (returns with “significant business income”) and those with auditor–adjusted wage and salary income exceeding 75 percent of auditor–adjusted total income (returns with “primarily wage income”). Summary statistics for the subsample with significant business income are reported in Table 1B and the latter in Table 1C. Based on the reasoning summarized above, we should expect that returns with primarily wage income will report a greater share of income than those with significant business income. We also would like to know whether those returns with primarily wage income, and therefore at greater risk of detection from misreporting income, use the deductions mode more or less intensively than those with significant business income.

As the third row section totals of Tables 1B and 1C show, returns with significant business income are more likely to understate income than those with primarily wage income—52 percent versus 43 percent, respectively. Furthermore, returns with significant business income understate taxable income more extensively than their primarily wage income counterparts. This is true in absolute terms ($5,350 versus $2,330) and as a fraction of auditor–adjusted AGI (9.2 percent versus 4.5). In contrast, returns with primarily wage income

<table>
<thead>
<tr>
<th>TABLE 1B</th>
<th>CROSS-TABULATION BY COMPLIANCE STATUS</th>
<th>SUBSAMPLE WITH SIGNIFICANT BUSINESS INCOME</th>
<th>(ALL DOLLAR FIGURES ARE AVERAGES)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Income</td>
<td>Understate</td>
<td>Correctly state</td>
<td>Overstate</td>
</tr>
<tr>
<td>Overstate</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percentage</td>
<td>20%</td>
<td>44%</td>
<td>36%</td>
</tr>
<tr>
<td>Income gap</td>
<td>$2,797</td>
<td>$1,003</td>
<td>$1,587</td>
</tr>
<tr>
<td>Deductions gap</td>
<td>$963</td>
<td>$0</td>
<td>$1,458</td>
</tr>
<tr>
<td>Taxable Income gap</td>
<td>$3,759</td>
<td>$1,003</td>
<td>$129</td>
</tr>
<tr>
<td>AGI</td>
<td>$52,992</td>
<td>$38,488</td>
<td>$70,756</td>
</tr>
<tr>
<td>Correctly state</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Percentage</td>
<td>13%</td>
<td>64%</td>
<td>23%</td>
</tr>
<tr>
<td>Income gap</td>
<td>$0</td>
<td>$0</td>
<td>$0</td>
</tr>
<tr>
<td>Deductions gap</td>
<td>$717</td>
<td>$0</td>
<td>$1,341</td>
</tr>
<tr>
<td>Taxable Income gap</td>
<td>$717</td>
<td>$0</td>
<td>$1,341</td>
</tr>
<tr>
<td>AGI</td>
<td>$51,058</td>
<td>$33,440</td>
<td>$56,317</td>
</tr>
</tbody>
</table>

| Understate | | | | |
| Percentage | 22% | 43% | 35% | 52% |
| Income gap | $6,250 | $3,668 | $6,432 | $5,205 |
| Deductions gap | $1,646 | $0 | $1,431 | $145 |
| Taxable Income gap | $4,604 | $3,668 | $7,863 | $5,350 |
| AGI | $65,316 | $45,396 | $69,702 | $58,307 |

| Column totals | | | | |
| Percentage | 19% | 50% | 32% | 100% |
| Income gap | $4,306 | $2,282 | $5,356 | $2,417 |
| Deductions gap | $3,440 | $0 | $4,232 | $201 |
| Taxable Income gap | $866 | $2,282 | $9,588 | $2,618 |
| AGI | $60,026 | $39,570 | $66,862 | $52,038 |

\*Income gap equals reported total income less corrected total income.
\*\*Deductions gap equals reported deductions (+ adjustments) less corrected deductions (+ adjustments).
\*\*\*Taxable income gap equals reported taxable income less corrected taxable income.
\*\*\*\*AGI equals total income less adjustments
wage income are more likely to overstate deductions than those with significant business income—39 percent versus 32 percent, respectively. Despite the apparently greater propensity of wage earners to employ the deductions mode, returns with significant business income, conditional on exaggerating deductions, understate taxable income by a greater average amount in absolute terms and as a fraction of auditor–adjusted AGI than their primarily wage income counterparts.

Tables 1A through C provide evidence that taxpayers use multiple modes of tax evasion and seem to do so in a manner consistent with the incentives created by third–party reporting of wage income and the lack of third–party reports in the case of business income. We would like to know whether the detection probability in a given mode has a positive effect on compliance in the targeted mode and an inverse effect on compliance in the untar-geted mode. To investigate this and other issues, we turn, now, to our multivariate analysis.

**EMPIRICAL ANALYSIS**

We assume that there are two potential modes of tax evasion: (1) total income reporting compliance and (2) deductions and adjustments (standard deduction) reporting compliance. Even when we assume fixed and independent detection probabilities, our theoretical model shows that increased enforcement effort in a given mode will influence compliance in both modes. Consequently, we assume that income and deductions reporting
compliance are simultaneously determined in our empirical model.

Accordingly, we assume taxpayer i’s income reporting compliance, denoted \( y_{ij} \), depends upon a vector of explanatory variables, \( X_{ij} \), and upon taxpayer i’s deductions and adjustments reporting compliance, denoted \( y_{ik} \), where subscript i denotes the individual and subscript j denotes the mode of evasion. Likewise, taxpayer i’s level of deductions reporting compliance depends upon a similar, though not identical, vector of explanatory variables, denoted \( X_{ik} \), and upon taxpayer i’s level of income reporting compliance. These relationships can be summarized as follows:

\[
\begin{align*}
   y_{ij} &= X'_{ij} B_j + \pi_j y_{ik} + \epsilon_{ij}; \\
   y_{ik} &= X'_{ik} B_k + \pi_k y_{ij} + \epsilon_{ik}.
\end{align*}
\]

We assume the error terms \( \epsilon_{ij} \) and \( \epsilon_{ik} \) have zero means and finite variances.\(^{20}\)

Our empirical model employs variables commonly used in the tax compliance literature. These include the last dollar marginal tax rate in each mode of evasion, total pre-tax income, proxies for probabilities of detection for each mode, and certain demographic information.\(^{21}\) In order to identify the parameters of the model \((B_j, B_k, \pi_j, \pi_k)\), we need at least one variable unique to each equation. Accordingly, we assume the detection probabilities in mode \( j \) are unique to equation [6], and the detection probabilities in mode \( k \) are unique to equation [7]. In addition, the last dollar marginal tax rate is likely to be endogenous as well.\(^{22}\) As described in greater detail below, we employ the first dollar marginal tax rate as an instrumental variable for the potentially endogenous last dollar rate.\(^{23}\)

**Construction of the Variables**

The following is a brief description of the construction of these variables. Column 1 of Table 2 provides summary statistics for these variables.

**Dependent Variables**

The log of the income gap is the natural logarithm of the maximum of 1.0 and auditor–adjusted total income less taxpayer reported total income. Total income includes wages; net business income from Schedule C (Sole Proprietorship), F (Farm), and E (Partnership, S Corporation, and rental income) activities; realized capital gains; and interest and dividend income; among others, excluding certain non–taxable sources, such as excluded dividends and capital gains.

The log of the deductions and adjustments gap is the natural logarithm of the maximum of 1.0 and taxpayer reported deductions and adjustments less auditor–adjusted deductions and adjustments. Deductions and adjustments include exemptions for self, spouse, and dependent children; adjustments; and itemized deductions or the standard deduction. As

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\(^{20}\) It is evident from Table 1A that the dependent variable is not symmetrically distributed. In fact, the data are highly skewed. Thus, we cannot assume that the errors are normally distributed. Since we have a rather large sample, we invoke the assumption that the t-ratios are asymptotically normally distributed. Given the skewness of the data, future work should explore econometric techniques that depend on the median as the measure of central tendency rather than the mean.

\(^{21}\) Note the lack of variation in the statutory penalty rate schedule does not permit us to estimate the effect of penalty rates on compliance. However, it is possible to vary penalty rates by using an experimental approach. See, for example, Cummings et al. (2001).

\(^{22}\) Identifying the tax price (see Feenberg (1987)) is always a concern with cross–sectional data. Therefore, although more recent TCMP data are available, we have decided to use the 1985 TCMP data because there is more cross–sectional variation in the tax price in pre–TRA 1986 data.

\(^{23}\) First–dollar marginal tax rates are obtained using auditor–adjusted data. Last–dollar marginal tax rates are calculated with taxpayer–reported data.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>---</td>
<td>-2.0541 (0.4689)</td>
<td>-8.9725 (0.5895)</td>
<td>-1.5856 (0.4495)</td>
<td>-8.9662 (0.5893)</td>
</tr>
<tr>
<td>Log of income</td>
<td>-3.382 (7.812)</td>
<td>-0.4163 (0.0505)</td>
<td>-0.0625</td>
<td>-0.4064 (0.0489)</td>
<td>-0.0621 (0.0619)</td>
</tr>
<tr>
<td>Log of the income tax price</td>
<td>-0.344 (0.208)</td>
<td>-6.3303 (0.3018)</td>
<td>(—)</td>
<td>-6.1505 (0.2974)</td>
<td>(—)</td>
</tr>
<tr>
<td>Log of the deductions tax price</td>
<td>-0.259 (0.206)</td>
<td>(—)</td>
<td>-3.0526 (0.3651)</td>
<td>(—)</td>
<td>-3.0136 (0.3648)</td>
</tr>
<tr>
<td>Married</td>
<td>0.693 (0.461)</td>
<td>1.4360 (0.1004)</td>
<td>0.0721 (0.1226)</td>
<td>1.4010 (0.9999)</td>
<td>0.0580 (0.1225)</td>
</tr>
<tr>
<td>Family size</td>
<td>2.593 (1.422)</td>
<td>0.1497 (0.0307)</td>
<td>0.0570 (0.0361)</td>
<td>0.1396 (0.0305)</td>
<td>0.0551 (0.0361)</td>
</tr>
<tr>
<td>Age</td>
<td>45.81 (16.63)</td>
<td>0.2345 (0.0113)</td>
<td>0.1287 (0.0137)</td>
<td>0.2273 (0.0111)</td>
<td>-1.310 (0.1364)</td>
</tr>
<tr>
<td>Age-squared*10^{-3}</td>
<td>2.375 (1.630)</td>
<td>-2.5269 (0.1162)</td>
<td>-1.3305 (0.1366)</td>
<td>-2.2514 (0.1142)</td>
<td>(—)</td>
</tr>
<tr>
<td>Wage share</td>
<td>0.669 (1.197)</td>
<td>-1.7527 (0.0561)</td>
<td>(—)</td>
<td>-1.7704 (0.0559)</td>
<td>(—)</td>
</tr>
<tr>
<td>Interest and dividend share</td>
<td>0.154 (0.443)</td>
<td>-1.0514 (0.1261)</td>
<td>(—)</td>
<td>-1.0760 (0.1261)</td>
<td>(—)</td>
</tr>
<tr>
<td>Dummy for sole prop income</td>
<td>0.314 (0.464)</td>
<td>0.0288 (0.0061)</td>
<td>(—)</td>
<td>0.0287 (0.0608)</td>
<td>(—)</td>
</tr>
<tr>
<td>Dummy for farm income</td>
<td>0.095 (0.294)</td>
<td>2.1722 (0.1093)</td>
<td>(—)</td>
<td>2.2367 (0.1079)</td>
<td>(—)</td>
</tr>
<tr>
<td>Dummy for rental income</td>
<td>0.249 (0.433)</td>
<td>2.3397 (0.0778)</td>
<td>(—)</td>
<td>2.3113 (0.0774)</td>
<td>(—)</td>
</tr>
<tr>
<td>Dummy for partnership income</td>
<td>0.183 (0.387)</td>
<td>-0.1960 (0.0892)</td>
<td>(—)</td>
<td>-0.2282 (0.0887)</td>
<td>(—)</td>
</tr>
<tr>
<td>Deductions share</td>
<td>0.228 (0.244)</td>
<td>(—)</td>
<td>3.3043 (0.2020)</td>
<td>(—)</td>
<td>3.3070 (0.2020)</td>
</tr>
<tr>
<td>Dummy for itemizer status</td>
<td>0.586 (0.444)</td>
<td>(—)</td>
<td>4.8206 (0.1328)</td>
<td>(—)</td>
<td>4.8268 (0.1328)</td>
</tr>
<tr>
<td>Log of the deductions gap</td>
<td>2.093 (7.510)</td>
<td>-0.1340 (0.0376)</td>
<td>(—)</td>
<td>(—)</td>
<td>(—)</td>
</tr>
<tr>
<td>Log of the income gap</td>
<td>3.382 (7.812)</td>
<td>(—)</td>
<td>-0.0173 (0.0065)</td>
<td>(—)</td>
<td>(—)</td>
</tr>
<tr>
<td>Sigma</td>
<td>---</td>
<td>6.0960 (0.0335)</td>
<td>6.5734 (0.0445)</td>
<td>6.0974 (0.0335)</td>
<td>6.5743 (0.0445)</td>
</tr>
<tr>
<td>Number of observations</td>
<td>42,811</td>
<td>42,811</td>
<td>42,811</td>
<td>42,811</td>
<td>42,811</td>
</tr>
</tbody>
</table>

Notes:
Standard deviations of the estimated coefficients are provided in parentheses.
Sigma—Maximum likelihood estimate of the parameter \( \sigma \) of the normal distribution, which is defined by the parameters of the model (the vector of slope coefficients and \( \sigma \)). Although a ‘t–ratio’ can be computed for \( \sigma \), a test of the hypothesis that \( \sigma \) equals zero \((0.0)\) is meaningless.

Income gap equals reported total income less corrected total income.

Deductions gap equals reported deductions (+ adjustments) less corrected deductions (+ adjustments).

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reported in Table 2, the average values for the log of the income and deductions gaps are 3.382 and 2.093, respectively.

The log of the income (deductions) gap is widely adopted in the empirical literature, but, in the context of this study, there is a drawback to this specification. Specifically, the logarithmic specification masks some legitimate tax evasion strategies, namely strategies 4 and 7 above. Nevertheless, the double log specification allows for potential non-linear effects and allows us to compare our results to those in the literature. Since the data are truncated at zero, we estimate the model using Two Stage Tobit (2ST).

**Independent Variables**

Log of total income is defined as the natural logarithm of auditor–adjusted pre–tax total income. As previously noted, total income includes wages, net business income from Schedule C, E, and F activities, capital gains (taxable and non–taxable), interest and dividend income (taxable and non–taxable), and other miscellaneous sources. In our sample, average auditor–adjusted pre–tax income is equal to approximately $62,407.

The log of the last dollar tax price is the natural logarithm of one (1.0) minus the last dollar marginal tax rate. The last dollar marginal tax rate is calculated using data as reported by the taxpayer. We believe that the theoretically appropriate marginal tax rate for decision making is the last dollar rate which includes the effect of any anticipated evasion activities undertaken by the taxpayer. Thus, the last dollar tax rate is potentially endogenous. The first dollar marginal tax rate excludes the effect of any tax evasion activities. Therefore, the first dollar rate is exogenous and, consequently, a proper instrument for the last dollar rate. A number of features of the tax code, such as the phase–out range of the earned income tax credit and the self–employment tax, drive a wedge between the marginal tax rate on income and deductions. Accordingly, we compute two variants of the first and last dollar marginal tax rates: (1) an income variant by adding one hundred dollars to wage income and (2) a deductions variant by adding one hundred dollars to itemized deductions. As shown in Table 3, the average of the log of the last dollar tax price of income (deductions) in our sample is –0.344 (–0.259), which corresponds to a marginal tax rate of 29.1 (22.8) percent.

**Scope for Misreporting Tax Attributes**

We also require measures for the perceived risk of detection in each mode. However, the preferred measure, the subjective probability of detection in each mode, is inherently unobservable. Many researchers have struggled with the difficulty of finding appropriate proxies for detection probabilities when estimating income reporting compliance equations.

24 Since the logarithm of non–positive values does not exist, it is necessary to set such values equal to 1.0 before taking the logarithm. In particular, cases in which taxable income is overstated are set equal to zero and, therefore, appear to be compliant; however, such cases are presumably the result of honest mistakes. Assuming honest mistakes are white–noise reporting errors, truncating the data in this manner means that all understatements of taxable income are treated as deliberate, when, in fact, some may be the result of honest mistakes. This approach also obscures more sophisticated evasion strategies, such as understating or overstating income and deductions.

25 A detailed tax calculator is used to construct four variants of the marginal tax rates. The calculator accounts for both the statutory rate schedule and many implicit tax rates that arise from special features of the tax code. Last–dollar marginal rates are obtained by adding one hundred dollars to reported wages for the income variant and by adding one hundred dollars to charitable contributions in the case of the deductions variants. First–dollar marginal rates are calculated by adding one hundred dollars to auditor–adjusted wage income (charitable contributions).
<table>
<thead>
<tr>
<th>Variable</th>
<th>Audit Class 1</th>
<th>Audit Class 2</th>
<th>Audit Class 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>IGAP</td>
<td>DGAP</td>
</tr>
<tr>
<td>Constant</td>
<td>—</td>
<td>−7.431</td>
<td>−7.905</td>
</tr>
<tr>
<td></td>
<td>(0.905)</td>
<td>(1.145)</td>
<td>(1.022)</td>
</tr>
<tr>
<td>Log of income</td>
<td>9.530</td>
<td>0.243</td>
<td>−0.401</td>
</tr>
<tr>
<td></td>
<td>(0.962)</td>
<td>(0.107)</td>
<td>(0.133)</td>
</tr>
<tr>
<td>Log of the last dollar income tax price</td>
<td>−0.198</td>
<td>−3.177</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>(0.201)</td>
<td>(0.817)</td>
<td>(1.285)</td>
</tr>
<tr>
<td>Log of the last dollar deductions tax price</td>
<td>−0.128</td>
<td>—</td>
<td>−13.027</td>
</tr>
<tr>
<td></td>
<td>(0.101)</td>
<td>(1.285)</td>
<td>(0.140)</td>
</tr>
<tr>
<td>Wage share</td>
<td>0.733</td>
<td>−0.775</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>(0.446)</td>
<td>(0.196)</td>
<td>(0.850)</td>
</tr>
<tr>
<td>Interest and dividend share</td>
<td>0.179</td>
<td>−1.309</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>(0.528)</td>
<td>(0.272)</td>
<td>(0.321)</td>
</tr>
<tr>
<td>Dummy for sole prop income</td>
<td>0.103</td>
<td>4.268</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>(0.144)</td>
<td>(0.183)</td>
<td>(0.421)</td>
</tr>
<tr>
<td>Dummy for farm income</td>
<td>0.021</td>
<td>3.960</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>(0.144)</td>
<td>(0.346)</td>
<td>(0.207)</td>
</tr>
<tr>
<td>Dummy for rental income</td>
<td>0.109</td>
<td>3.968</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>(0.311)</td>
<td>(0.178)</td>
<td>(0.486)</td>
</tr>
</tbody>
</table>
### Table 3 (continued)

**Two-Stage Tobit Estimates of the Log of the Income and Deductions Gaps, by Audit Class**

| Variable                          | Audit Class 1 | | Audit Class 2 | | Audit Class 3 | |
|-----------------------------------|---------------|----------------|----------------------------|----------------|----------------|
|                                   | Mean | IGAP | DGAP | Mean | IGAP | DGAP | Mean | IGAP | DGAP |
| Dummy for partnership income      | 0.051 | 0.349 | —   | 0.341 | -0.050 | —   | 0.170 | -0.366 | —   |
|                                   | (0.221) | (0.248) |     | (0.474) | (0.126) |     | (0.376) | (0.122) |     |
| Deductions share                  | 0.128 | —   | 8.437 | 0.354 | —   | -0.042 | 0.207 | —   | 0.856 |
|                                   | (0.221) |     | (0.415) | (0.206) |     | (0.283) | (0.247) |     | (0.436) |
| Dummy for itemizer status         | 0.286 | —   | 3.090 | 0.936 | —   | 5.024 | 0.567 | —   | 6.984 |
|                                   | (0.452) |     | (0.225) | (0.245) |     | (0.274) | (0.496) |     | (0.296) |
| Log of the income gap<sup>a</sup> | 1.885 | —   | 0.102 | 3.146 | —   | -0.226 | 6.363 | —   | 1.08  |
|                                   | (2.879) |     | (0.072) | (3.556) |     | (0.063) | (3.610) |     | (0.164) |
| Log of the deductions gap<sup>b</sup> | 1.455 | 0.162 | —   | 3.144 | -0.366 | —   | 1.793 | -0.422 | —   |
|                                   | (2.701) | (0.065) |     | (3.351) | (0.113) |     | (2.923) | (0.061) |     |
|                                   | (0.066) | (0.097) |     | (0.058) | (0.054) |     | (0.036) | (0.106) |     |
| Number of observations            | 17,098 | 17,098 | 17,098 | 15,248 | 15,248 | 15,248 | 10,465 | 10,465 | 10,465 |

Notes:

Standard deviations of the estimated coefficients are provided in parentheses.

Sigma—Maximum likelihood estimate of the parameter $\sigma$ of the normal distribution, which is defined by the parameters of the model (the vector slope coefficients and $\sigma$). Although a ‘t–ratio’ can be computed for $\sigma$, a test of the hypothesis that $\sigma$ equals zero (0.0) is meaningless.

<sup>a</sup>Income gap equals reported total income less corrected total income.

<sup>b</sup>Deductions gap equals reported deductions (+ adjustments) less corrected deductions (+ adjustments).
We face the added burden of developing probabilities of detection for two modes of evasion. At first blush, objective audit probabilities would seem to be good candidates for the perceived detection probability. For obvious reasons, such information is closely held by the IRS. Furthermore, objective audit probabilities are often a function of reported tax attributes, particularly in the case of field audits, and, as such, are likely to be endogenous. If, for example, taxpayers believe that the tax authority audits taxpayers with unusually high ratios of itemized deductions relative to total income, then taxpayers that dramatically underreport income may choose not to report some valid deductions in order to reduce the probability of detection.

Rather than use objective audit probabilities, researchers have tended to use proxy variables that measure the scope for tax evasion on a return. Since wage income is subject to third–party reporting and withholding, taxpayers with a lower share of wages in AGI have a greater scope for understating income and a lower attendant probability of detection than those with higher shares. As such reasoning suggests, the IRS estimates that 99.5 percent of wage income is voluntarily reported for federal income tax purposes and much lower rates for business income, which is often not subject to third–party reporting (See Clotfelter (1983) and Joulfaian and Rider (1996)). We develop additional measures of the scope for evasion on a return by extending this logic to include other line items that are subject to third–party reporting. Specifically, we construct the following three proxies for the scope of evasion on a given return by using third–party reporting of particular line items.

*Wage share* is the ratio of auditor–adjusted wages and auditor–adjusted total income from taxable sources. *Interest and dividend share* is the ratio of auditor–adjusted interest and taxable dividends and auditor–adjusted total income from taxable sources. *Itemized deductions share* is the sum of auditor–adjusted deductions for home mortgage interest and state and local taxes as a proportion of auditor–adjusted total itemized deductions. If the taxpayer does not itemize, then this value is set equal to zero. Thus, by construction, this variable also distinguishes between itemizers and non–itemizers. Clearly, itemizers have a greater opportunity to misreport deductions with a lower attendant risk of detection than do non–itemizers.26

The wage and interest and dividend shares serve as proxies for the scope of evasion in the income reporting compliance equation, while the itemized deductions share performs this role in the deductions reporting compliance equation. Since wages and salary also are subject to withholding, while interest and dividends are not, we allow them to have different effects on compliance by entering them separately into the income reporting compliance equation. As shown in column 1 of Table 2, wage income is approximately 67 percent of auditor–adjusted total income, while interest and dividends constitute only 15.4 percent of auditor–adjusted total income. The deductions covered by third–party reporting are nearly 23 percent of auditor–adjusted itemized deductions and adjustments.

In addition, we include dummy variables for the presence of Schedule C (31.4 percent of the sample), Schedule F (9.5 percent), rental income (24.9 percent), partnership income (18.3 percent), and itemized deductions (58.6 percent). Since these sources of income are not subject to third–party reporting, there is a greater scope to misreport these sources of income and, therefore, these dummy variables serve as suitable proxies for the

26  Note that non–itemizers can still exaggerate exemptions, adjustments, and the number of standard deductions.
probability of detection or the scope for misreporting income.

**Demographic Variables**

In order to control for differing attitudes toward risk and tastes for evasion, we include three demographic variables that have been shown in the literature to influence tax compliance behavior (see footnote 15). These include an indicator variable for marital status (single versus married) and the number of dependents (family size). We also include the age of the primary taxpayer and age-squared to account for any non-linearity in this variable.27 As shown in Table 2, approximately 69.3 percent of the observations in our sample are married, the average number of dependents is 2.6, and the average age of the primary filer is approximately 46 years.

**EMPIRICAL RESULTS**

Now, we turn to a discussion of our 2ST estimates, using our sample of 42,811 returns. For the sake of comparison, we present both simultaneous and independent estimates of both equations. Specifically, the simultaneous estimates of the coefficients of the (log) income and deductions gap equations are reported in columns 2 and 3 of Table 2, respectively, and the estimated coefficients from the independent estimation of these two equations are reported in columns 4 and 5, respectively. The independent equations are also estimated by 2ST due to the potential endogeneity of the last dollar tax price.

We begin with a discussion of one of our most fundamental research questions. Do taxpayers use alternative modes of tax evasion as substitutes or complements? In the income gap equation (column 2), the estimated coefficient of the (log) deductions gap is equal to –0.134 and statistically significant at conventional levels. The corresponding elasticity is –0.08. In other words, a ten percent increase in deductions reporting compliance results in a decrease in income reporting compliance by 0.8 percent. Likewise, the estimated coefficient of the (log) income gap in the deductions gap equation (column 3) is equal to –0.017 and statistically significant at conventional levels. The corresponding elasticity is equal to –0.01; thus, increasing income reporting compliance by ten percent decreases deductions reporting compliance by 0.1 percent.

It is interesting to note that the elasticity of the income gap with respect to the deductions gap is eight times larger than the elasticity of the deductions gap with respect to the income gap. Thus, increasing income reporting compliance increases total compliance more than does increasing deductions reporting compliance. The estimates suggest that income and deductions reporting compliance are substitute modes of evasion. In other words, enforcement strategies that increase revenue through increased reporting compliance in one mode result in a partially offsetting reduction in revenue from deteriorating compliance in the other mode.

Focusing on the estimates for the log of the income gap equation, the log of the tax price is negative and statistically significant at conventional levels. In other words, there is an inverse relationship between the marginal tax rate and income reporting compliance. This is consistent with much of the empirical literature. The corresponding elasticity is –3.92; consequently, a ten percent increase in the tax price, or a decrease in the marginal tax rate, increases income reporting compliance by approximately 39 percent.

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27 Although taxpayers do not report their age on the tax form, we are able to obtain the necessary information to calculate the age of the primary filer from social security records.
The estimated coefficient of the log of total income is negative and statistically significant at conventional levels. The estimated elasticity is equal to –0.26, i.e., a ten percent increase in total income increases income reporting compliance by 2.6 percent. This estimate is at odds with some previous findings in the literature. However, it is interesting to note that we also get a negative and statistically significant estimated coefficient for the log of total income in the independent case (column 4). Therefore, the sign of this estimate does not appear to be simply an artifact of the simultaneous specification.

In addition to the deductions gap variable discussed above, the wage share and interest and dividend share serve as proxies for the probability of detecting income misreporting. The estimated coefficients for these variables are negative and statistically significant at conventional levels. In other words, increasing the share of income subject to third-party reporting reduces the scope for misreporting income and, therefore, increases compliance. This is consistent with previous findings reported in the literature.

We also include a set of indicator variables for different types of business income, which are not subject to third-party reporting or withholding. As expected, the estimated coefficients for the indicator variables for sole proprietorship, farm, and rental income are positive and statistically significant at conventional levels. Interestingly, the indicator variable for partnership income is negative and statistically significant at conventional levels. Since a partnership is a pass through entity for tax purposes, misreporting partnership income requires collusion among all the partners. The difficulties and risks of forming such conspiracies may explain the negative coefficient. In short, these variables have the expected signs and are consistent with much of the literature.

Furthermore, the estimated coefficients on married, number of dependents, and age are positive and statistically significant at conventional levels, while the estimated coefficient of age-squared is negative and statistically significant at conventional levels. The combined effect of age and age-squared is positive for those under 46 years old, and negative otherwise. Thus, married taxpayers, taxpayers with dependents, and older taxpayers are less compliant. Again, this is generally consistent with results reported in previous studies. Finally, the simultaneous estimates (column two) from the income gap equation are completely consistent with the independent estimates (column four). This is somewhat reassuring in that neglecting the simultaneity between the two modes of evasion does not appear to seriously bias the estimates.

Turning to the estimates from the log of deductions gap equation, the results are very similar to those obtained for the income gap equation. Specifically, the log of the tax price and log of total income have a negative and statistically significant (at conventional levels) effect on deductions reporting compliance. The estimated elasticities of the tax price and income are equal to –1.65 and –0.03, respectively. Though marital status and number of dependents are positive, they are not statistically significant at conventional levels. As in the income gap equation, age and age-squared are positive and negative, respectively, both are statistically significant at conventional levels. The combined effect of the two is positive for those less than 48 years old, and negative otherwise. Itemizer status serves as a proxy variable measuring the scope for misreporting deductions. In other words, there is greater scope to misreport deductions when the

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28 Clotfelter (1983) also obtains negative and statistically significant estimates in some audit classes, while Feinstein (1991) reports a positive though statistically insignificant estimate in his pooled regressions.
taxpayer itemizes. As one would expect, the estimated coefficient on this variable is positive and statistically significant at conventional levels.

Surprisingly, the estimated coefficient on deductions share is positive and statistically significant at conventional levels. In other words, there is a negative relationship between the share of itemized deductions subject to third-party reporting and deductions reporting compliance. Again, the simultaneous estimates, which are discussed above and reported in column 3 of Table 2, are consistent with the independent estimates reported in column 5.

To control better for unobserved heterogeneity in detection probabilities in our sample, we stratify the sample into three IRS-defined categorical audit classes and re-estimate equations [4] and [5]. The audit classes are defined according to audit probabilities, based on total positive income and other criteria thought by the IRS to reflect the extent of tax underreporting. Needless to say, information on audit probabilities is closely held by the IRS; but, suffice it to say that returns in a given audit class are more or less equally likely to be subject to an IRS audit. The descriptive statistics and estimated coefficients for these subsamples are reported in Table 3. The descriptive statistics for audit classes 1, 2, and 3 are reported in columns 1, 4, and 7, respectively. We see that audit classes 1 and 2 have much higher values for wage share (73.3 and 84.2 percent, respectively) than does audit class 3 (31.1 percent). On the other hand, audit classes 2 and 3 have a much higher proportion of itemizers (93.6 and 56.7 percent, respectively) than does audit class 1 (28.6 percent). Furthermore, audit class 3 has a much higher proportion of Schedule C filers (77.8 percent) than do audit classes 1 and 2. Finally, it is worth noting that the log of the last dollar tax price is significantly larger for audit class 2 than for the other two.

The audit class results are not entirely consistent with those for the entire sample. More specifically, the log of the income gap is negative and statistically significant in the deductions gap equations (columns 3, 6, and 9) for audit class 2 (column 6), positive and statistically significant for audit class 3 (column 9), and statistically insignificant at conventional levels in audit class 1. Similarly, the log of the deductions gap in the income gap equation (columns 1, 4, and 8) is negative and statistically significant for audit classes 2 and 3, and positive and statistically significant in audit class 1. The log of the tax-price is negative and statistically significant in 4 out of 6 cases. Generally speaking, the estimated coefficients for the variables that measure the scope for evasion in each mode have the anticipated signs and are statistically significant. More specifically, the estimated coefficients of wage share and interest share are negative and statistically significant for audit classes 1 and 2. As in Table 2, however, the estimated coefficient on the deductions share is positive and statistically significant for audit classes 2 and 3. Likewise, the dummy variables for business income are generally positive and statistically significant, except for partnership income, which is either negative and statistically significant, or statistically insignificant.

It is interesting to note that for audit classes 1 and 2, the mean of the log of the income gap tends to be rather small. This may be due to the relatively high wage share of these two subsamples relative to that of audit class 3. In other words, as the wage share increases, the scope for evasion decreases and income reporting compliance increases. Similarly, as the scope for misreporting deductions increases, deductions reporting compliance should decrease. Indeed, the proportion of itemizers is much higher for audit class 2 compared to that of the others, and the mean of the log of the deductions gap is larger.
IMPLICATIONS FOR TAX ENFORCEMENT

We began this study by noting that multiple modes of tax evasion complicate the task of tax administration. We find from our theoretical model that increased enforcement effort in a given mode has an ambiguous effect on compliance in the targeted mode as well as the untargeted mode. Our empirical model suggests that increased enforcement effort in a given mode increases compliance in the targeted mode and decreases compliance in the untargeted mode. In order to gauge the relative magnitudes of these opposing effects, we use our empirical model to simulate two alternative enforcement strategies.

We begin by changing the share of income subject to third–party reporting and withholding by ten percent relative to the mean value in our data. This policy change leads to a 1.8 percent increase in income reporting compliance and a 0.1 percent decrease in deductions reporting compliance. The net effect of a ten percent increase in income subject to third–party reporting is a 1.1 percent increase in taxable income reporting compliance. Since we have estimated the model in logs, it is important to note that the effect of, say, a 40 percent change in covered wages will be somewhat less than four times the change induced by a ten percent change. In other words, the effect is not linear.

Next, we consider the effect on compliance of changing a taxpayer from an itemizer to one that simply claims the standard deduction. This simulation is conducted by evaluating both estimated equations at the mean values of our data, with the itemizer dummy variable set equal to 1.0. We compare the resulting predictions with those obtained when we set the itemizer dummy variable equal to zero. The impact of this strategy is quite dramatic. The deductions gap is eliminated, while the income gap increases by ten percent. The net effect of this policy change is a 40 percent increase in taxable income reporting compliance. Recall that since the left–hand–side of both equations is in logs, the estimated effect of a given change in itemizer status is not linear.

CONCLUSION

In this paper, we examine the theoretical and empirical implications of accounting for multiple modes of evasion on taxpayer compliance behavior. In our theoretical model, we find that the effect of increased enforcement effort in a given mode has an ambiguous effect on compliance in the targeted mode as well as the untargeted mode. Thus, it is not possible to predict whether taxpayers perceive alternative modes of evasion as substitutes or complements. Clearly, the answer to this question has important implications for tax administration. If increased enforcement effort in a given mode increases compliance in the targeted mode and the modes are treated as substitutes, then the revenue gain from increased compliance in the targeted mode will be offset by decreasing compliance in the untargeted mode. If the two modes are complements, the revenue effect of increased compliance in one of the modes reinforces that in the other.

We estimate an empirical model of tax evasion with two modes of evasion, using data from the IRS’s 1985 TCMP, to address this important issue. Three results in particular are worth noting. First, we report empirical evidence that income and deductions reporting compliance are substitutes. In other words, increased enforcement effort in a given mode increases compliance in the targeted mode; but, it is partially offset by deteriorating compliance in the other mode. Second, our estimates suggest that there is a negative relationship between the tax rate and income (deductions) reporting compliance. Finally, we use our estimates to simulate two enforcement strategies: (1) increasing the share of income subject to third–party
reporting and withholding by ten percent; and (2) eliminating itemized deductions. Both policies lead to increases in taxable income reporting compliance; but, eliminating itemized deductions appears to be particularly effective. Given the potentially significant implications of multiple modes of evasion for tax administration, further research in this area is justified.

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**APPENDIX**

We assume that individuals attempt to maximize expected utility (EU) as defined by the following expression:

\[
[A–1] \quad EU = (1 - P_1)(1 - P_2)U(Z_1) + P_2 U(Z_2) + P_1(1 - P_2)U(Z_3) + P_2 U(Z_4),
\]

where \( Z_1 = Y - T + E_1 + E_2 \); \( Z_2 = Y - T - \theta_1 E_1 + E_2 \); and \( Z_3 = Y - T - \theta_1 E_1 - \theta_2 E_2 \).

We assume that the detection probabilities are fixed and independent, or:

\[
\frac{\partial P_1}{\partial E_1} = \frac{\partial P_1}{\partial E_2} = \frac{\partial P_2}{\partial E_1} = \frac{\partial P_2}{\partial E_2} = 0.
\]
The necessary conditions for an interior maximum of [A–1]:

\[ [A–2] \quad \frac{\partial EU}{\partial E_1} = (1 - P_1)[(1 - P_2)U'(Z_1)] \\
+ P_2U'(Z_1) - \theta P_1[(1 - P_2)U'(Z_1)] \\
+ P_2U'(Z_1) = 0 \]

\[ [A–3] \quad \frac{\partial EU}{\partial E_2} = (1 - P_1)[(1 - P_2)U'(Z_1)] \\
- P_2\theta U'(Z_2) + P_1[(1 - P_2)U'(Z_1)] \\
- P_2\theta U'(Z_2) = 0 \]

From [A–2] and [A–3], we can derive the following condition for the existence of an interior solution \((E_1 > 0 \text{ and } E_2 > 0)\) of [A–1]:

\[ \Theta_i < 1 - \frac{P_i}{P_j} < \Theta \left( \frac{U'(Z_i)}{U'(Z_j)} \right) \]

where \(i = 1, 2\).

Proof: From [A–2] and [A–3], it is straightforward to show the following:

\[ \frac{\partial EU}{\partial E_1} \bigg|_{E_i = 0} > 0 \text{ implies } \Theta < \frac{1 - P_i}{P_i} \text{ and } \frac{\partial EU}{\partial E_1} \bigg|_{E_i = Y} < 0 \text{ implies } \frac{1 - P_i}{P_i} < \Theta \left( \frac{U'(Z_i)}{U'(Z_j)} \right) \]

where \(i = 1, 2\) and \(\frac{U'(Z_i)}{U'(Z_j)} > 1\).

Sufficient conditions for an interior maximum of [A–1]:

We assume that the second–order conditions are satisfied by the concavity of the expected utility function. The second–order conditions of a local maximum of [A–1] are given as follows:

\[ [A–4] \quad \frac{\partial^2 EU}{\partial E_1^2} = (1 - P_1)[(1 - P_2)U''(Z_1)] \\
+ P_2U''(Z_1) + \theta P_1[(1 - P_2)U''(Z_1)] \\
+ P_2U''(Z_1) < 0; \]

\[ [A–5] \quad \frac{\partial^2 EU}{\partial E_2^2} = (1 - P_1)[(1 - P_2)U''(Z_2)] \\
+ P_2\theta U''(Z_2) + P_1[(1 - P_2)U''(Z_2)] \\
+ P_2\theta U''(Z_2) < 0; \]

\[ [A–6] \quad \frac{\partial^2 EU}{\partial E_1 \partial E_2} = (1 - P_1)[(1 - P_2)U''(Z_3)] \\
+ P_2\theta U''(Z_3) - P_1[(1 - P_2)\theta U''(Z_3)] \\
- P_2\theta U''(Z_3) \geq 0. \]

We proceed by deriving and discussing our main comparative static results.

The effect of changing income on tax compliance:

By totally differentiating [A–2] and [A–3] with respect to \(E_i, Y\), and then solving the resulting system of equations simultaneously, we obtain the following expressions for the effect of income on tax compliance:

\[ [A–7] \quad \frac{dE_1}{dY} = - \left[ \frac{A \cdot EU_{1a} - B \cdot EU_{1b}}{M} \right] \text{ and } \]

\[ [A–8] \quad \frac{dE_2}{dY} = - \left[ \frac{B \cdot EU_{1b} - A \cdot EU_{1a}}{M} \right], \]

where

\[ A = (1 - P_1)[(1 - P_2)U''(Z_1) + P_2U''(Z_1)] \\
- \theta P_1[(1 - P_2)U''(Z_1) + P_2U''(Z_1)] \geq 0; \]

\[ B = (1 - P_1)[(1 - P_2)U''(Z_3) - \theta P_2U''(Z_3)] \\
+ P_1[(1 - P_2)U''(Z_3) - \theta P_2U''(Z_3)] \geq 0; \]

and

\[ M = \left[ \frac{\partial^2 EU}{\partial E_1^2} \right] - \left[ \frac{\partial^2 EU}{\partial E_1 \partial E_2} \right] > 0. \]

Since we know that \(M > 0\) from the concavity of the expected utility function, we only need to expand the numerator of [A–7] to gain greater insight into the nature of the ambiguous ef-
fect of income on tax compliance in mode 1. Expanding the numerator of [A–7], we obtain the following expression:

\[
\begin{align*}
[A–9] & \quad B \times EU_{12} - A \times EU_{22} = \\
& \quad - P_2(1 - P_1)(1 - P_1)(1 + \theta_2)(U''(Z_j))(1 - P_2; P_{12}; (1 - P_2)(1 - P_1) + \theta_2)(U''(Z_j)) - P_2(1 - P_1)(1 - \theta_1)(U''(Z_j)) \leq 0.
\end{align*}
\]

Since the algebraic signs of expressions \(D\) and \(EU_{12}\) are unknown, the effect of changing \(P_1\) has an ambiguous effect on tax compliance in the targeted mode as well as the untargeted mode. At first, this result may not seem intuitive. We can gain greater insight into this expression by showing that [A–10] can be decomposed into income and substitution effects.

Using [A–7] and [A–11], we can rewrite [A–10] as follows:

\[
[A–12] \quad \frac{dE_1}{dP_1} = \left[ \frac{D}{C} \frac{dE_2}{dP_1} \right] + \left[ \frac{C}{A} \frac{dE_2}{dY} \right] + \left[ \frac{D}{C} \frac{EU_{12}}{M} \right],
\]

In words, expression [A–12] says that the effect of increasing \(P_1\) on the targeted mode \(E_1\) can be decomposed into a substitution effect \((dE_2/dP_1)\) and an income effect \((dE_2/dY)\). The sign of the substitution effect is ambiguous, depending on whether the two modes of evasion are substitutes or complements. As previously discussed, the sign of the income effect is ambiguous. The sign of the third bracketed expression is also ambiguous. Given the uncertainty about the signs and magnitudes of these quantities, the effect of changing the probability of detection in mode 1 on tax compliance in mode 1 is ultimately an empirical question.

Expression [A–10] also can be rewritten in terms of an income and substitution effect as follows:

\[
[A–13] \quad \frac{dE_2}{dP_1} = \left[ \frac{C}{D} \frac{dE_1}{dP_1} \right] + \left[ \frac{D}{B} \frac{dE_2}{dY} \right] + \left[ \frac{C}{D} \frac{EU_{12}}{M} \right].
\]
Again, the algebraic sign of \([A-13]\) is ambiguous because the signs of the bracketed expressions are unknown.

The effect of changing a detection penalty on tax compliance:

By totally differentiating the first–order conditions \([A-2]\) and \([A-3]\) by \(E_1, E_2, \) and \(\theta_1,\)

and solving the resulting system of equations simultaneously, we obtain the following expressions for the effect of a change in the penalty on compliance:

\[
\frac{dE_1}{d\theta_1} = -\left[ \frac{F \cdot EU_{11} - G \cdot EU_{12}}{M} \right],
\]

and

\[
\frac{dE_2}{d\theta_2} = -\left[ \frac{G \cdot EU_{12} - F \cdot EU_{12}}{M} \right],
\]

where \(F = P_1(1 - P_2)\theta_1U'(Z_3) + P_2(1 - P_2)\theta_2U'(Z_4)\)

and \(G = P_1(1 - P_2)\theta_1U''(Z_3) + P_2(1 - P_2)\theta_2U''(Z_4).\)

Since the algebraic signs of expressions \(G\) and \(EU_{12}\) are unknown, the effect of increasing \(\theta_1\)

has an ambiguous effect on compliance in the targeted mode \((E_1)\) as well as the untargeted mode \((E_2).\) Again, we conclude that the effect of penalties on tax compliance is an empirical question.